Dimensions, a Math Movie

Aurélien ALVAREZ
Maître de conférences
Université d'Orléans
aurelien.alvarez@univ-orleans.fr

Jos LEYS
Ingénieur, Visualisation mathématique
Antwerp, Belgium

jos.leys@pandora.be
http://www.josleys.com

Abstract : Dimensions is a two-hour animated movie, aimed at a broad audience, produced by Jos Leys, Étienne Ghys and Aurélien Alvarez. The notion of 'dimension' in the mathematical sense is explained in 9 chapters: 'Dimension 2' talks about location on a sphere and stereographic projection.. 'Dimension 3' explains how 2- dimensional creatures can imagine 3-dimensional objects, which is an introduction to 'Dimension 4' where we show how we, as 3-dimensional creatures, can imagine 4- dimensional objects. Next is a visual introduction to complex numbers, leading in to the Hopf fibration as an example of 4-dimensional math. As an epilogue we show a formal proof of a geometric theorem related to stereographic projection. Through this film, the authors wanted to show that math does not need to be 'dry', but that math can produce beautiful imagery. In order to reach as wide an audience as possible, the film is a non-profit project. The DVD has a low price, and the films can be downloaded free of charge from an internet site featuring additional information on the subjects of the film. Furthermore, the film has a 'Creative Commons' license, which allows copying of the film (provided there is no commercial gain). The film is available in 8 commentary languages and 20 languages for the subtitles.

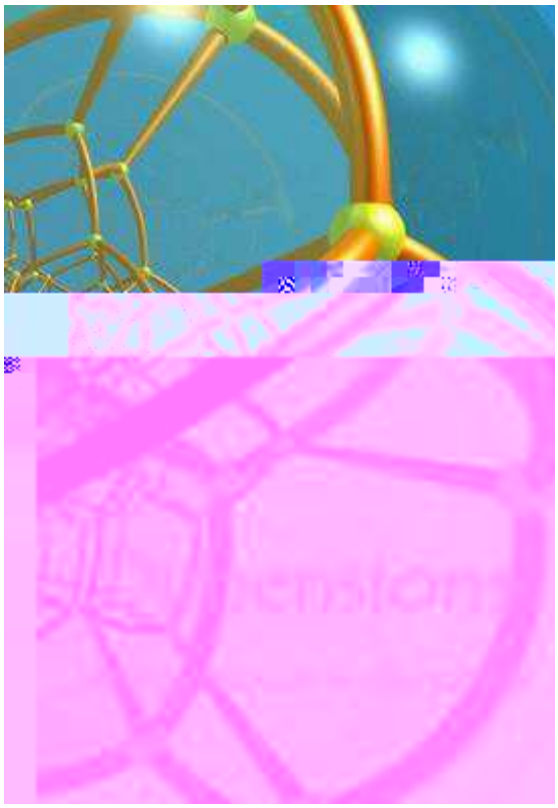

Fig. 1. DVD cover

1. Introduction : the start of a film project

Early 2006, Jos Leys and Étienne Ghys, "directeur de recherche" at the École Normale Supérieure" at Lyon, made their acquaintance on the internet : Ghys was looking for some images to illustrate a general-public talk he was giving, and stumbled upon Leys'website [1] where there are a large number of mathematics based images and animations. Later that year Ghys was to give a plenary talk at the 2006 ICM in Madrid, and he proposed to collaborate on the making of a series of images and animations to illustrate this talk. An intensive exchange of emails followed, with Ghys' instructions and Leys' test images, culminating in a successful plenary talk on "Knots and Dynamics"[2,3].

During this project, on numerous occasions, Ghys had to explain advanced math concepts to Leys who is a non-mathematician, and from this came the idea for a film : explaining a set of math concepts in simple terms to as wide an audience as possible.

2. The scenario

We chose to explain the notion of “dimensions” in the mathematical sense. The choice followed naturally from the work on “Knots and Dynamics” where a lot of the math was 4-dimensional, and furthermore we thought that higher dimensions would have a lot of appeal. The film consists of 9 chapters of 13 minutes each, which is a good length of time for eventual classroom use.

‘Dimension 2’ talks about location on a sphere and stereographic projection. (We are making use of the latter in all the subsequent chapters.) ‘Dimension 3’ explains how 2-dimensional creatures can imagine 3-dimensional objects, which is an introduction to ‘Dimension 4’ where we show how we, as 3-dimensional creatures, can imagine 4-dimensional objects. As 4-dimensional objects we show a series of 4-dimensional polytopes such as the tesseract or the 120-cell. Next is a visual introduction to complex numbers, leading in to the Hopf-fibration as an example of both complex numbers and 4-dimensional math. As an epilogue we show a formal proof of a geometric theorem related to stereographic projection.

There are some still images from the film in the paragraphs below, in no particular order.

3. The production

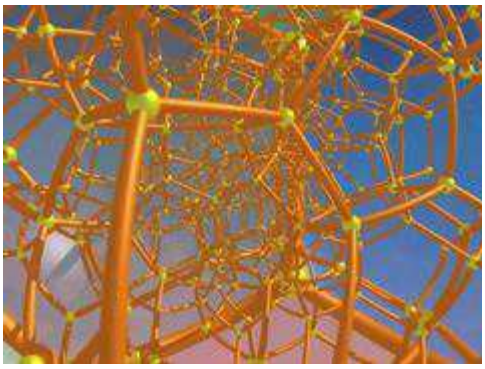

Fig. 2. 4-dimensional polytope

All of the images and animations were done in Povray [4], a free raytracing program. At 25 frames per second, we needed to render a total of about 175,000 images sized at 800*600 pixels. The present version of Povray does not allow multiple core processing, so the rendering of the images is relatively slow. We managed to put to work a whole series of computers at the “Pôle Scientifique de Modélisation Numérique”, the computing centre at the École Normale Supérieure in Lyon where both Ghys and Alvarez resided. Each computer was given a series of images to compute, which were then collated by Alvarez to be turned into Quicktime animations using Final Cut Pro, a video editing program.
The music was chosen from royalty free sources on the internet, and from compositions by Florent Ghys, a nephew of Étienne Ghys.

4. A non-profit project

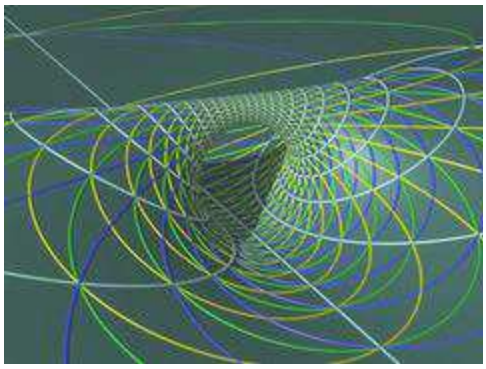

Fig. 3. Dupin cyclide

In order to reach as wide an audience as possible, we quickly concluded that this had to be a non-profit project, so the film could be accessible to all. This means selling the DVD at a low cost which mainly covers expenses, willingness to give out DVD's for free to appropriate organisations (as an example we provided free copies for the participants at a Math Olympiad, and to a convention of French math teachers) and a simple licensing scheme. We chose a Creative Commons [5] license which allows copying without commercial gain with proper attribution.

The ENS at Lyon graciously provided a small grant to cover our startup costs, mainly for the production of the DVD's. (There was little or no interest from other institutions or from publishers.)

In order to keep costs as low as possible, we used mainly free or open source software for the entire project, and depended heavily on unpaid volunteers for commentary voices in different languages, and for making translations. In fact we attribute the success of the film project for a large part to the availability in multiple languages. The film is available with commentary voice in 8 languages and subtitles are available in 20 languages.

5. The website [6]

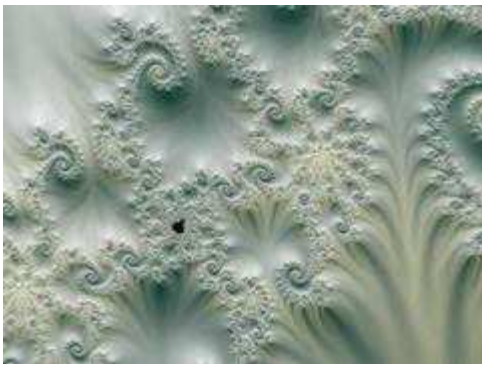

Fig. 4. Mandelbrot fractal

We felt from the start that we needed a website for this project. Firstly because it is just about the only way we had to let the world know that the film exists at all. For this aim we could just have made a one-page site with a small description of the film and a button to buy the DVD. However we felt that we could do a lot more. There is now on the website a page for every chapter of the film with additional information on the different topics, pages where the film can be downloaded freely (several mirror sites across the world are available) in various formats (Quicktime high resolution, Ipod format), directly or by Torrent, pages where the film can be watched online, and of course a page where the DVD can be ordered. (The price of 10€ covers the cost of the DVD as well as worldwide shipping.) Through the hard work of dozens of volunteers, the website is now available in 10 languages (French, English, Dutch, German, Arabic, Russian, Chinese, Japanese, Spanish and Portuguese.)

6. Results

Fig. 5. Stereographic projection

Since the day of the opening mid-2008, the website has been visited close to a million times from just about every country in the world. The number of downloads of the film is difficult to estimate because of the propagation via Torrents, but we see that the site at Lyon has a bandwidth of a few hundred gigabytes per day. To date we have had 12,000 DVD's produced of which about two thirds have been sold, and one third given away.
In June 2010, the film received the "Prix d'Alembert", a bi-annual price given by the French Mathematical Society to what they judge is the best mathematics divulgation project of that period. More importantly, we still receive enthusiastic reactions from all over the world on a daily basis.

7. Conclusion

We believe that with his project, we have proven that it is possible to produce a good math divulgation film with very limited resources, provided there is a team of enthusiasts willing to spend a considerable amount of time. Success is only guaranteed with a good team. In our case it consisted of two mathematicians and an engineer, and we found that this collaboration works very well.
In fact, this same team has plans for a new project.

[1] Mathematical Imagery : *http://www.josleys.com*

[2] Étienne Ghys on "*Knots and Dynamics*" at the 2006 ICM in Madrid :
http://www.mathunion.org/Videos/ICM2006/muster.php?ghys2006
[3] Étienne Ghys and Jos Leys, *Lorenz and modular flows : a visual introduction.* Feature column of the AMS :
http://www.ams.org/samplings/feature-column/fcarc-lorenz
[4] See http://www.povray.org .
[5] Creative Commons : Attribution – Non commercial – No derivatives 3.0
http://creativecommons.org/licenses/by-nc-nd/3.0/
[6] *http://www.dimensions-math.org*