Nonlinear Musical Analysis and Composition

Renato Colucci, Gerardo R. Chacón and Juan Sebastian Leguizamon C.

Abstract We discuss the application of Nonlinear time series analysis in the context of music analysis. We comment the results presented in [4] and give some ideas for further investigation (see [5]). Moreover we show how these techniques can be used to produced original music with both artistic and pedagogical purposes.

1 Introduction

The methods of non linear time series analysis has been widely used in studying many natural and social phenomena (see for example [7], [11] and [9]). The most important tool is Takens' theorem (see [12]) that lets us reconstruct the whole phase space by considering the data in a proper m-dimensional Euclidian space. If we denote by $\{x_i\}_{i=0}^N$ the original data set then the m dimensional vectors, called m-histories, are constructed in the following way:

$$h_1 = (x_0, \dots, x_{m-1})$$
$$\dots$$
$$h_{N-m+1} = (x_{N-m+1}, \dots, x_N)$$

The dynamics on the pseudo-phase space $\mathscr{H} = \{h\}_{i=1}^{N-m+1}$ is diffeomorphic to the dynamics on the true attractor of the system. Then analyzing the data on the embedding space is it possible to obtain the estimation of many important quantities such

Renato Colucci
Pontificia Universidad Javeriana, Carrera 7 N. 43-82 Bogotá,
e-mail: renatocolucci@hotmail.com
Gerardo R. Chacón
e-mail: grchacon@gmail.com
Juan Sebastian Leguizamon C.
e-mail: sebasgta@gmail.com

as the dimension of correlation or Liapunov exponents. In [8] and [4] a way to apply these techniques has been proposed in the context of musical analysis (see section 2). In the present paper we continue the implementation of nonlinear techniques in the context of music. In particular we give some examples of application of prediction algorithm to simulate a musical styles (see section 3). In section 4 we show that prediction algorithm can also be used to produce original music. In particular we suggest a method of random interpolation of a set of m-histories with another set corresponding to two different composition (here we chose Prelude of Suite I for Cello by J,S, Bach and Sequenza IXb for Alto Saxophone by L. Berio). The new set of m-histories will be used to make prediction and the result will be some mixture of the two compositional styles. There are many possibilities to apply this methods to compose original music both for artistic and pedagogical purposes. In the last section we give some remarks and suggestions for future investigations.

2 Discussion of previous results

In [4] the authors analyzed three different compositions and discussed the technical difficulties while applying these techniques in the context of musical analysis. The compositions analyzed, Prelude of Suite n. 1 for cello solo (1720-1721) by J.S. Bach (1685-1750) (see [13]), Syrinx (1913) by C. Debussy (1862-1918) (see [6]) and Tenor Saxophone Solo from Acknowledgement (from the Album A Love Supreme, 1964) by J. Coltrane (1926-1967) (see [3]) are all compositions written for an instrument. The authors of the paper made an identification of a solo musical composition (see [8]) with a time series and apply some time series techniques in order to analyze them. As a summary of the results of [4], the three different compositions written using different styles such Baroque Counterpoint (Bach), Free Modern Composition (Debussy) and Modal Jazz (Coltrane), satisfy the following inequalities concerning embedding dimension (m) and correlation dimension (D):

$$m_{Debussy} \leq m_{Bach} < m_{Coltrane}, \tag{1}$$

$$D_{Debussy} < D_{Bach} < D_{Coltrane}. \tag{2}$$

This suggest that the music of John Coltrane can be described by using more patterns/variable with respect to music of Debussy but looking at the inequality regarding Liapunov exponents (L):

$$L_{Bach} < L_{Coltrane} < L_{Debussy}, \tag{3}$$

we guess that the patterns of Debussy are arranged in a more unpredictable way with respect to that of Coltrane's.

Then it is natural to ask if the above reasonable results are due only to the great difference between the three compositions or if these methods really work in general cases. It could be interesting to ask if it is possible to catalogue music by these nonlinear techniques depending on styles, genres, composers, etc.

Here we present, as a first approach, the analysis of the whole Suite No. 1 in G major, BWV 1007 from ”Six suites for Cello by Johann Sebastian (1717-1723). The structure of the movements of the suite are the following:

- Prelude;
- Allemande;
- Courante;
- Sarabande;
- Minuet;
- Gigue.

In table 2 we represent the values of the standard deviation (σ), of the embedding dimension (m), correlation dimension (D) and Liapunov Exponents (L) of the six movements of the first suite. We observe that with the exception of the fourth movement, the other movements share similar values of the analyzed quantities. A deeper analysis on the problem of cataloguing music can be find in [5] where the authors use different techniques such as recurrence analysis and pattern analysis.

Movements	σ	**m**	**D**	**L**
1	5.62476483	7	2,405810844	0,245919883
2	5,59381701		2,626908577	0,194686888
3	5,33603512		2,276816981	0,12682068
4	4,47777764		1,635210943	0,059480429
5	5,531770331		2,045760793	0,160438678
6	4,662153419		2,429508864	0,125400802
Mean Value	5,2043863920		2,2366695	0,15212456

3 Prediction algorithm

The most important goal of the nonlinear time series analysis is to make predictions in order to understand the future behavior of a complex system. In the context of music analysis it is interesting to ask if it has sense to consider a prediction algorithm for a musical composition. In our opinion those algorithms could be used to simulate a musical style or to produce new music.

We give an example of that for the music of Coltrane, a complete study of problem of prediction can be found in [5].

We consider the following algorithm (see [1])

$$y_{t+1}^m = \sum_{i=1}^{N} \left[\hat{y}_{k+1}^m - \hat{y}_k^m + y_t^m \right] \omega_k(y_t^m, \hat{y}_k^m), \tag{4}$$

where y_t^m is the last $m-$history of our data set, y_{t+1}^m is the $m-$history that we want to predict, the points

$$\hat{y}_k^m \in B_\varepsilon(y_t^m), \quad k = 1, \ldots, N, \tag{5}$$

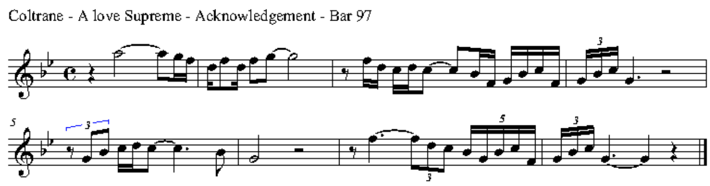
are the neighbors of y_t^m contained in the neighborhood B_ε and $\hat{y}_{k+1}^m$ are the next points of $\hat{y}_k^m$. The weight functions are given by the following expression

$$\omega_k(a,b) = \frac{K_h(\|a-b\|)}{\sum_{k=1}^{N} K_h(\|a-b\|)}, \tag{6}$$

with the Gaussian Kernel:

$$K_h(x) = \frac{1}{h} K\left(\frac{x}{h}\right), \qquad K(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \tag{7}$$

Once a set of $m-$histories has been produced it is possible to construct the one dimensional prediction by considering only the first element of each $m-$history.
In figure 3 we represent a portion of the Coltrane's solo from bar 97 to bar 104 while



in figure 3 we present a prediction with $\varepsilon = 30$ and $h = \frac{1}{2}$.

We consider the algorithm of prediction only for the tones and put almost all the original values of the tones (for prediction for both tones and values see [5]). We observe that the most used scale by Coltrane is the Pentatonic of G minor while in the prediction all the predicted tones belong to the Pentatonic scale of C minor which is very close to that of G minor, moreover Coltrane uses this pentatonic scale during the continuation of his solo. These simple examples show that prediction algorithm also work in the context of music.

4 Original Music Production

In this section we give an example of how to use these prediction algorithm to create new music. The possibilities of the algorithms have only the limits of author's creativity. We propose a method based on the interpolation of one composition with another. We consider the Sequenza IXb by L. Berio for Alto Saxophone and again The prelude of the first Cello Suite by J.S. Bach. We cut the Sequenza at part D (included) since this is an homogeneous part (see [10]) and the embedding dimension results to be 5. Then we consider the 5-histories of the Prelude, we note that the embedding dimension is 7. We interpolate randomly the 5-histories of the Sequenza with the 5-history of the prelude in the following way: we consider the first 5-histories of Sequenza and the algorithm randomly decides to insert the first 5-history of Bach or to continues with the second 5-history of Sequenza. We note that this procedure mixes the 5-history without changing the order in which the histories of Bach and Berio appear. When the new set of 5-history is constructed we are ready to make a prediction. Since almost all the tones of the Prelude have the same value of 16th note, for simplicity in this case we predict only the pitch and put all 16th note as in the prelude. A simple way to introduce variations in the rhythm, without using prediction algorithms for the duration of the tones, is to randomly assign values to the pitches. This method is more suitable for contemporary music, while for classical music it would be necessary to use more restrictions. It is possible to use also irregular groups in the style of Sequenza but the algorithm that assigns values should need some constraints. More examples and discussion on prediction of the values can be find in [5].
In figure 1 below we represent the result of the random interpolation (in a five dimensional embedding space) of the Sequenza with the Prelude using $h = 0.5$ and $\varepsilon = 30$. It is interesting to note that if we want to change the roles of the two compositions we have to change the embedding space. We randomly interpolate the 7-histories of the Prelude with 7-histories of the Sequenza, the result is presented in figure 2 below. Again we use $h = 0.5$ and $\varepsilon = 30$.

The production of new music will require more investigations and the experimentation of musicians. We remark that it is possible to interpolate a composition with original (written) or random material, or in the contrary, we could start from some original or random material and interpolate. This method would work also for live performances in which, due to the random interpolation, a different musical sheet could be produced at each concert.

5 Conlusion

In the present work we have discussed the results of [4] and gave some previews of the investigations about cataloguing and simulating musical styles which are the main topics analyzed in the forthcoming paper [5]. Moreover we give some examples of how these methods can be used in a variety of ways to produce original

Fig. 1 Random interpolation of the Sequenza IXb using the Prelude of Suite I.

music. Another technique from the nonlinear time series analysis that we consider could be useful for cataloguing and producing original music is given by the pattern recognition algorithms. In particular, we consider the machine learning method to recognize patterns (see for example [2]) should be explored in this setting and combined with the other techniques.

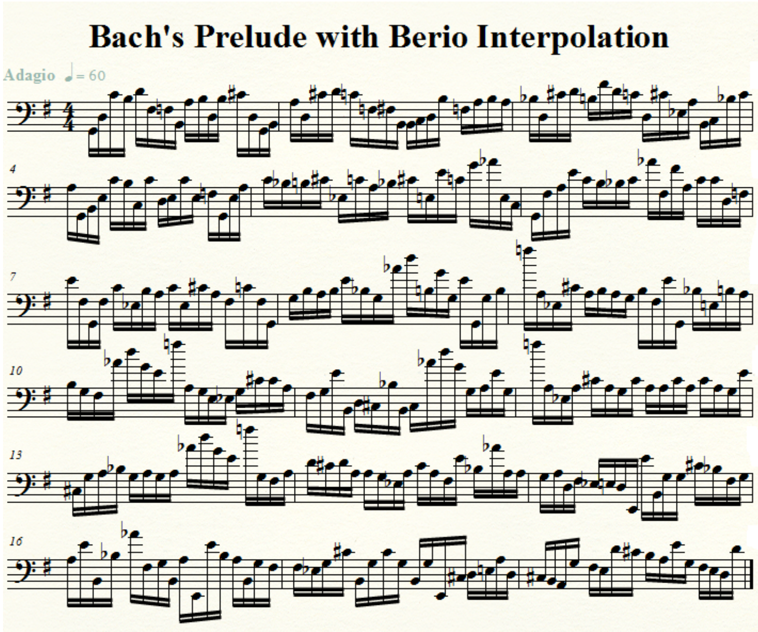


Fig. 2 Random interpolation of the Prelude of Suite I using Sequenza IXb.

References

1. A. Basharat and M. Shah, *Time series prediction by chaotic modeling of nonlinear dynamical systems*. Computer Vision, 2009 IEEE 12th International Conference, 2009.
2. C. Bishop. *Music: Pattern Recognition and Machine Learning*. Springer Verlag, 2006
3. J. Coltrane. *A Love Supreme: Tenor Saxophone*. Hal Leonard, 2003.
4. Renato Colucci, Gerardo R. Chacón and Sebastian Leguizamon C. *Some Ideas on Nonlinear Musical Analysis*. Applied Mathematical Sciences, Vol. 7, 2013, no. 26, 1283-1301.
5. Renato Colucci, Gerardo R. Chacón and Sebastian Leguizamon C. *Cataloguing and simulating music by nonlinear techniques*. Preprint.
6. C. Debussy. *Syrinx, pour Flûte seule*. J. Jobert, 1927.
7. A. Facchini, C. Mocenni, N. Maewan, A. Vicino and E. Tiezzi. *Nonlinear time series analysis of dissolved oxygen in the Orbetello Lagoon (Italy)* Ecological modelling 203, 2007.
8. C. Madden. *Fractals in Music*. High Art Press, Salt Lake City 2007 2nd edition.
9. R. Perli and M. Sandri *La ricerca di dinamiche caotiche nelle serie storiche economiche: una rassegna*. Note Economiche del Moten dei Paschi di Siena, Anno XXIV n. 2 (1994).
10. F. Sbacco *Appunti del corso di ansalisi musicale*. Unpublished, 2008.
11. S. Sun and X. Xing. *The Research of the Fractal Nature between Costs and Efficacy in the Brain Vascular Disease*. Journal of Applied Mathematics Volume 2012, Article ID 171406, 12 pages doi:10.1155/2012/171406.
12. F. Takens. *Detecting Strange Attractors in Turbulence*. Lecture Notes in Math. Springer, New York, 1981.
13. A. Winold *Bach's Cello Suites: Analyses and Explorations (Vol. 1 & 2)* . Indiana University Press, 2007.