Structure of Visualization and Symmetry in Iterated Function Systems

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Abstract. Principles and practices of visualization have always been valuable tools in all fields of research. The formation of representations plays a key role in all aspect of science. Prior research in the area of visualization demonstrates that representation of data hold great potential for enhancing comprehension of abstract concepts and greatly benefit collaborative decision-making and project performance. The purpose of this study is to examine the effectiveness of integrating various methodologies in the production of a visually coherent proposition. The author explores the dynamic of visualization in the digital environment and brings together elements of fractal topology, optical distortions and color theory components in an esthetic statement. The background and information selected for this purpose is based on specific cognitive processes, neurological and topological researches developed at the turn of the XX century by various experts in the field of investigative science: mathematicians Cantor and Sierpinski, neuroscientists Hermann, Hering, Wundt and other analytical minds such as Itten and Kandinsky that, at a particular time in history, demonstrated the similarities and overall cohesion of an intellectual and scientific discourse that helped articulate the technological and esthetic world we know today.

Image1. Cantor deconstructed

1. Introduction

Georg Cantor was a prominent mathematician who mapped out a fundamental theory in mathematics at the turn of the XX century. He established the importance of one-to-one correspondence between sets. The Cantor set theory is particularly well suited for visualizing and experimenting with mathematical topics such as iterated-function systems, similarity dimension, complex numbers, connectedness, and topological groups.

Topology is a major area of mathematics concerned with spatial properties. It emerged through the development of concepts such as space, dimension, and transformation. An iterated function system can be defined as an attractor formed by the union of a finite number of contraction or expansion mappings.

Some well-known generalizations of the Cantor set can be found in the Hausdorff's dimension similarity principle and the Sierpinski's square and triangle named after its author.

Sierpinski's iteration occurs when an object or image is transformed by a set of affine alterations to produce a new image. The new image is then modified by the same affine conversion to produce another new image.

At the particular time (mid 1800 - mid 1900) mathematicians were mapping those various theories, authorities in other disciplines such as psychological and physiological sciences were investigating optical aberration. Ewald Hering, Ludimar Hermann produced templates that had uncanny visual likeness to the work of Cantor or Sierpinski. Also to be noted, this time also saw the reevaluation of traditional 2 dimensional art paradigms with the arrival of cubism, constructivism and the many movements that lead to a radical reinterpretation of the esthetic environment. All elements very relevant to the object of this study.

The author's background being into fine art, and visual communication, the reader needs to be made aware that precision and exactitude in the field of mathematic may be at time only tentative. For more scientific rigor on fractal and chaos theory, one is encouraged to explore the work of more qualified experts in Laplacian mathematic, the papers of Pelander [1] and Barlow & Bass [2] on the solvability of differential equations of open subsets of the Sierpinski gasket, and other professional publications in the field of cognitive and biopsychology science.

2. Methodology

This experiment evolved around the following 3 elements:

1. A mathematical part to revisit fundamental concepts of set theory, symmetry and recursive function in topological spaces.
2. A technical part to develop graphic visualizations in a digital environment.
3. A subjective and artistic part to infuse various esthetic interpretations with the help of alternate color propositions.

2.1 Data collection - Mathematical iteration systems

According to Falconer [3], one of the essential features of a fractal is that its Hausdorff dimension generically strictly exceeds its topological dimension. Because they appear similar at all levels of magnification, fractals are often considered to be infinitely complex. Mandelbrot [4] and Julia sets offer today extraordinary complex visualizations made possible through the expression of computerized technology. However it is to be noted that the outcome, while technically sound, often ends up defeating a major component of visual statements because of the mechanical and repetitive nature of its expression: the esthetic environment seems to require for most some level of uncertainty well within the tissue of what contributes to human perception of reality.

Waclaw Sierpinski was the mathematician who at the turn of the century mapped visually the Cantor set theory in simple visual terms. He first became interested in set theory when he came across a theorem that stated that points in the plane could be specified with a single coordinate. Two popular visualizations that emerged from his research are known today as the Sierpinski gasket (triangle) and the Sierpinski carpet (square). A key element that made it a natural choice for this study on the effectiveness of a message initiated on a flat 2D surface is the fact that the Sierpinski iteration originates and progresses at a straight angle where most commonly rendered fractals are based on diagonal and curve progression.

2.2 Graphic manipulation

Vector graphics store lines and shapes as mathematical formulae. They provide accurate visual rendition and produce an image scalable to any size and detail. All compelling factors when keeping open multiprocessing output options and transfer mathematical information with exactitude. In the words of Dr. Konrad Polthier [5]

> "Mathematical visualization driven by computer graphics has proven to be a successful tool for mathematicians to investigate difficult mathematical problems"

2.2.1 Vector outline

Adobe Illustrator is a sophisticated vector based graphic software. It allows the user to explore and visualize mathematical relationships in terms of 2D line and precision drawing. It was selected as the best available tool to translate specific mathematical concept in visual terms and in particular for a system that transforms a sequence of symbols into a unique set of points in the 2-dimensional space. The Sierpinski square pattern requires that one starts with a solid square and divide it into 9 smaller congruent squares. The original square is scaled 8 times by a factor 1/3. Measurement standards were adjusted to the metric system to make calculation easier.

To solve the issue afferent to the identification of the squares in the progression, a grid 20% black down to 80%, was created and filled accordingly, starting with the 1 mm square (80% black) down to 20% for the 8th iteration. (Fig. 1)

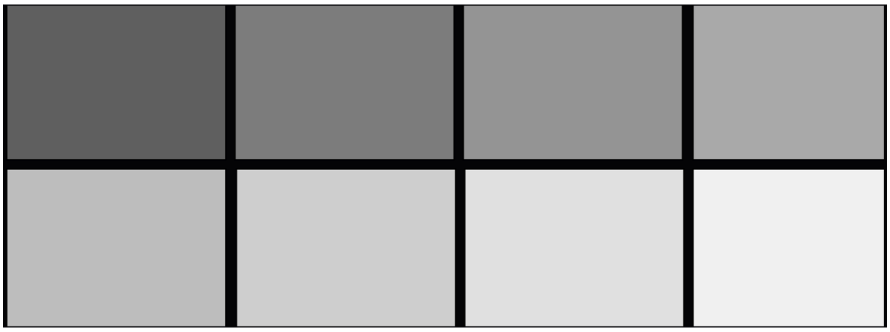

Fig. 1

Cartesian coordinates and topological dimensions defined by Sierpinski' s contemporaries Lebesgue (1903) and Hausdorff (1918) set the original measurement (point in space) as zero. For all practical purpose, this was not viable information to start from in the particular environment in which the demonstration was developed. A unit of one was chosen as the starting point. The total virtual canvas size would be 2187 mm or 2, 187 m., a size one assumed the processing power of the computer could handle. [10]

Moving to plate 243 mm, slight changes of x y / w h positioning started to be noticed and dimension of the smaller squares, (600.0007 mm turned into 600.009 mm in a simple copy/paste step) even after locking all elements. The computer system started to slow considerably during rendering. Also it was noted that the center square of plate 243 (Fig. 2, Fig. 2-b) filled with 40% black disappeared in the background because of the hue interference of too many squares 1- 3 and 9.

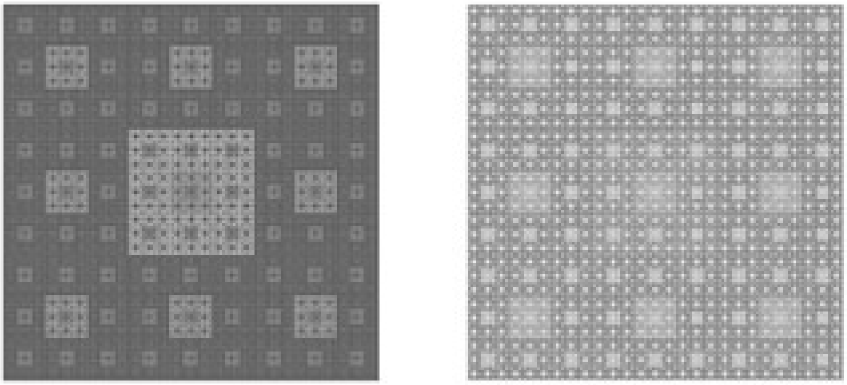

Fig. 2

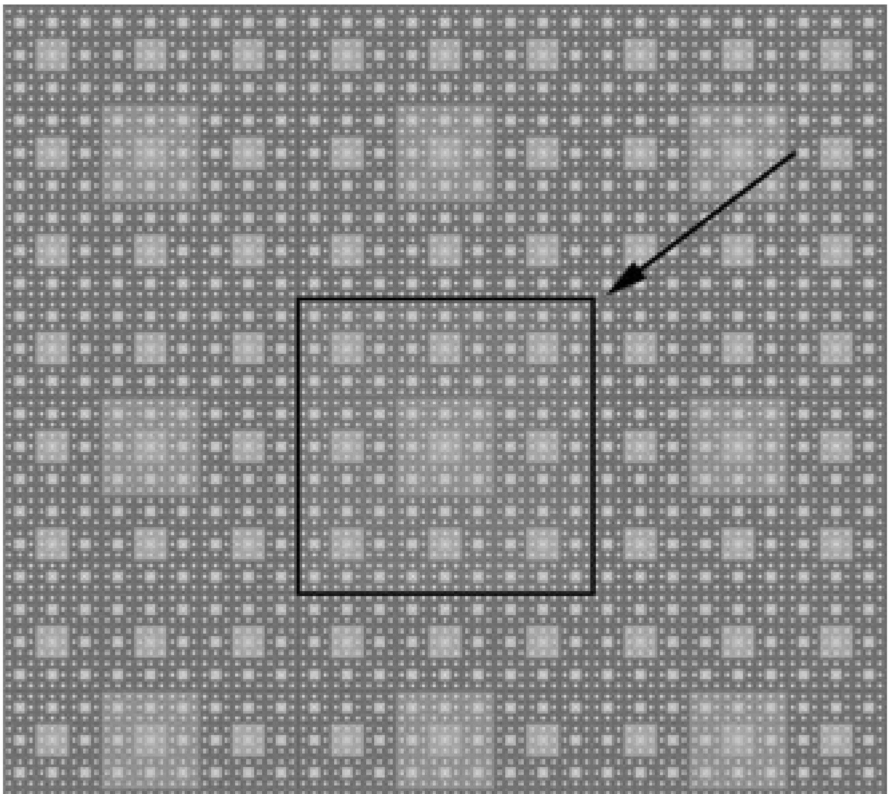

Fig. 2b

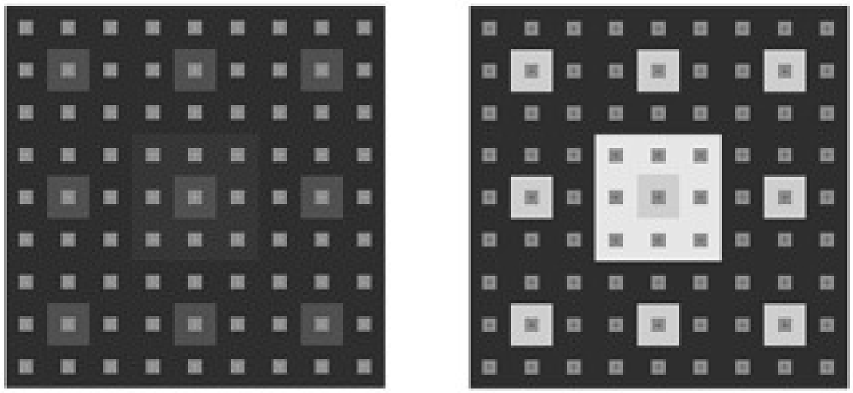

Fig. 3

To be consistent with the premises of the demonstration and bring the progression to 2187, it was decided to proceed with a reverse elimination of square-layers 1, 3, and 9 mm, except in the larger squares. (Fig. 3)

The end result shows intriguing optical similarities with symmetry patterns and tiling mentioned in CP Bruter expose in Saverne, [6] and that were developed over centuries in various civilizations across the world. (Fig. 4)

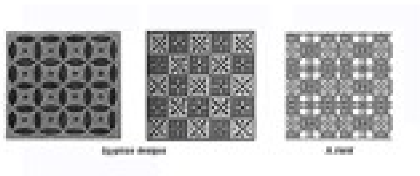

Fig. 4

A fact made all the more prevalent by the monochrome tone of the visualization that brings up the design element of the statement. (Fig. 5)

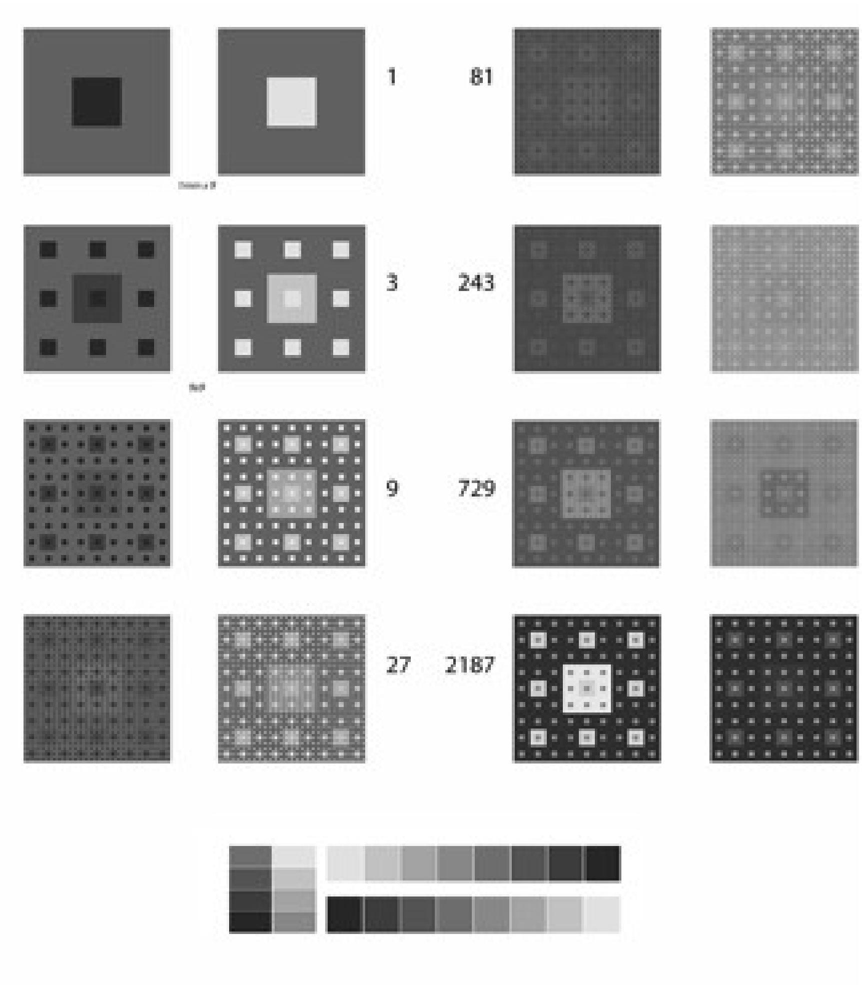

Fig. 5

2.2.2 Optical Illusion

Can we really trust what our eyes see?

An interesting property of an iteration in a two dimensional environment is that it is most likely to take the viewer off balance both for cultural and physiological reasons: anchor and points of reference are arbitrary and the determination of objects dynamic and elements of perspective idiosyncratic. The history of art is an ongoing experiment on the mapping of this particular challenge. It also outlines the extent and limitation of communicating on a flat surface. Science for the best part of the last two hundred years has brought a deep, universal, objective understanding to intuitive exploration of this universe made over centuries by talented artists in various cultures, worldwide. [7]

In the following stage, we attempted to reconcile and experiment in simple terms the findings of various disciplines involved in the exploration of human perception as well as the more subjective element of esthetic interpretation. Plate 2187 was transferred in a graphic editing program (Photoshop 10.00) where selected elements were extracted, blended, and recomposed as needed in new visualizations.

Below is a brief descriptive of the process

Hermann grid:

Dark patches appear in the street crossings, except the ones that you are directly looking at. If you look around in the neighboring figure you will notice the appearance and disappearance of black dots at the crossings. (Invoked to explain Florida's election problems in 2001: "Count the black dots, recount to confirm…")

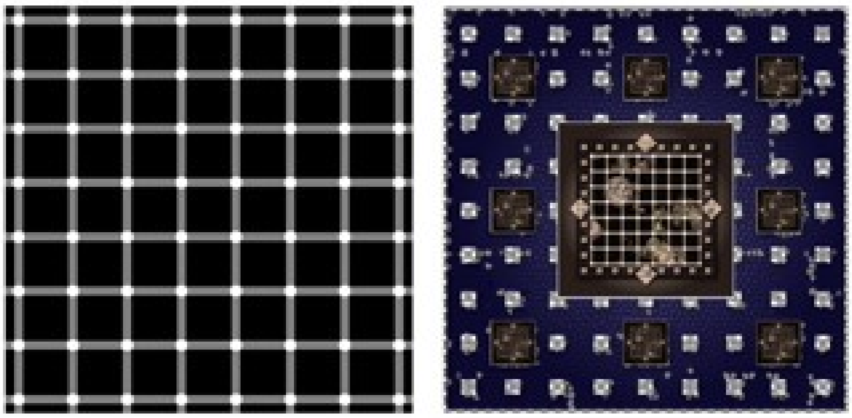

Fig. 6

The Hering illusion:

Discovered in the 19th century by German physiologist Ewald Hering. Two straight lines appear curved or bowed in the context of intersecting lines with orientations that change progressively. Hiding the oblique lines from the view will reveal the fact that the horizontal bars are equally straight.

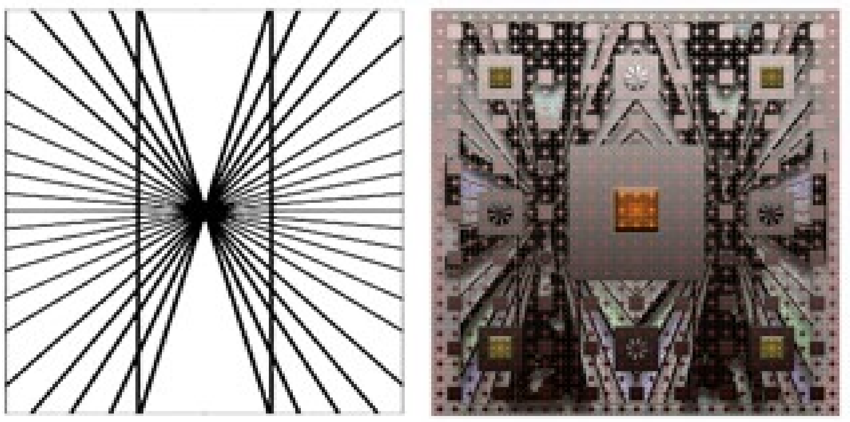

Fig. 7

The Wundt illusion:

Considered the founder of experimental psychology, Wilhelm Wundt introduced cognitive principles to the psychology community in the late 1880s. He was the first one to report on the phenomenon. The overestimation of the sub tense of acute angles and the underestimation of obtuse ones was first reported first by Wundt.

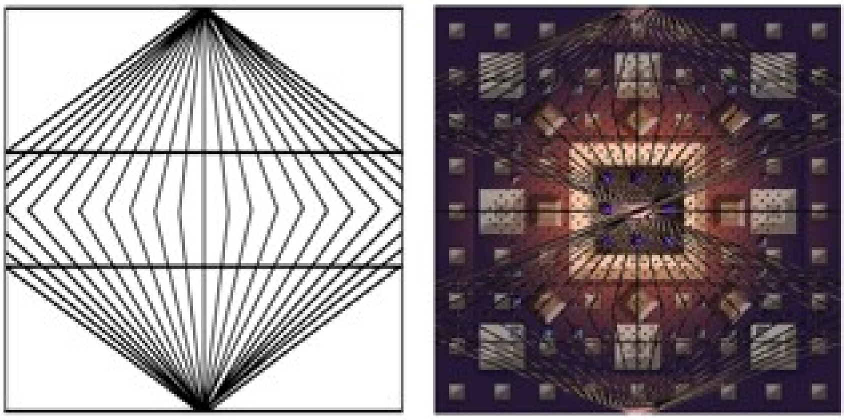

Fig. 8

The Aitken wheel:

In homage to Robert Grant Aitken astronomer, director of the Lick observatory, president of the University of California. Minor planet (3070) Aitken is named in his honor as well as a Moon crater 2 on the far side.

From another Aitken: Concentrating on the wheel center for a few minutes will make the wheel spin.

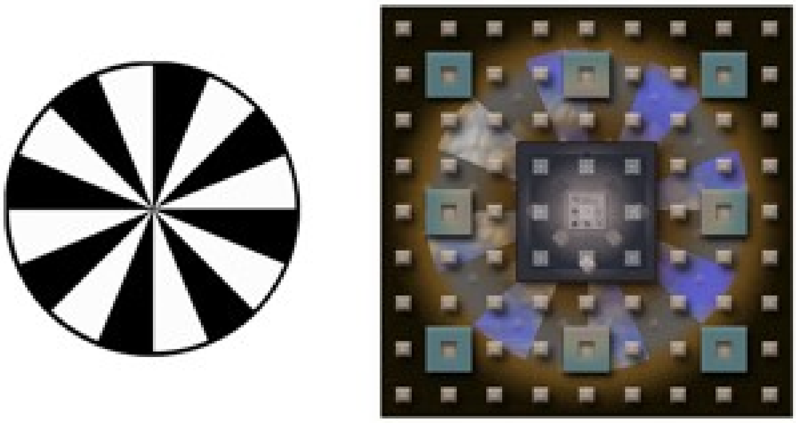

Fig. 9

The Ebbinghaus

Identical objects are perceived as unequal in size when objects of a different size surround them. This kind of assimilation illusion is also associated with the name of Franz Joseph Delboeuf, a Belgian philosopher. The copies of the same shape - circles and rings in our case - appear to inherit properties of their environment. Two equal circles - one inside a bigger circle, the other containing a smaller circle - seem to have different sizes.

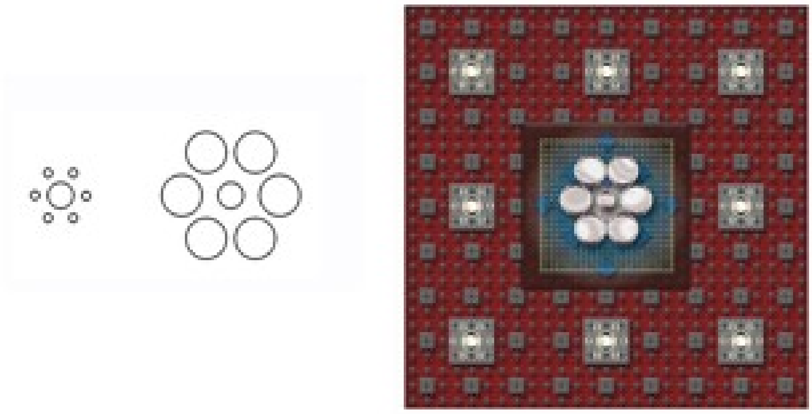

Fig. 10

2.2.3 Color scheme

Up to this point, the demonstration had been conducted strictly in a terms of black and white and shade of gray. Grayscale images allow to concentrate on measuring the intensity of light and the electromagnetic spectrum of any given frequency. It helps focus in precision mapping and objective rationalization of effect for best possible outcome.

In the early 1900's, Johannes Itten [8], color instructor at the Bauhaus and trained in psychoanalytic theory developed a color wheel that took into consideration the subjective feeling that's associated with objective color, and the psychic and emotional values of colors (Fig. 5). Itten's color wheel was used as a reference to expand this project and include another layer of visual interpretation.

Fig. 11

This work was made considerably easier by a very dynamic color tool from the Adobe Lab, the Kuler application. (Fig. 6) Its algorithm allows the user to create and test color combinations in real time and download them on the workspace as needed.

Fig. 12

Below is an example of the progression from black & white to color:

Fig. 13

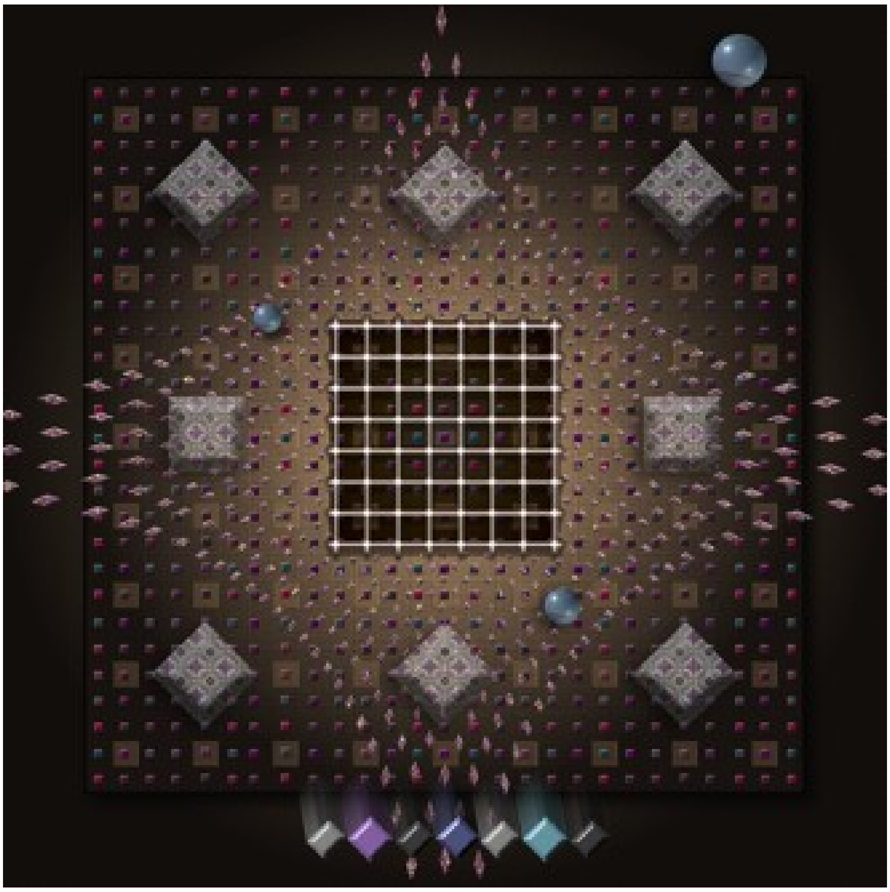

Image 2

2.3 Dissemination

This part of the expose is relatively succinct for a simple reason: the technological context in which we communicate is fast changing. Specific mechanical information that may be appropriate today could fast become obsolete in tomorrow environment.

However one compelling element legitimates mention of this stage of production: works of art as well as works of science are meant to be shared. Whatever the technology at hand, the ultimate goal of any endeavor of that nature is to be shared and benefit as many as possible so they can learn from the many blunders and missteps and hopefully build from some of the inspiration it provides.

2.3.1 Multimedia

The goal of developing a multimedia presentation was dual: summarize the experiment in an attractive presentation and use various techniques to enhance key points of the demonstration in visual terms. The final product was encoded for QuickTime viewing – QuickTime being a media viewing platform available on most computers today.

2.3.1 – a. Visual component

The Apple Keynote interface is among the most prevalent forms of persuasive technology used today in all fields of communication. Its powerful engine and multiple effects allow authors to maximize their effort and reach out effectively targeted audience. The progression of the experiment was extracted in a series of single slides that were combined with appropriate transitions and effects available in the program toolbox.

2.3.1 – b. Audio track

Professional animators believe that 70 percent of a moving-images presentation impact comes from the sound track. Sound guides the rhythm of the presentation; helps emphasize specific part and keep the viewer attention focused in the progression. The soundtrack background was provided by a Mikael Kott, a classically trained artist and remixed in an audio software to fit the particular of the presentation.

2.3.2. Deployment

The emergence of the Internet and other tools of modern communication have radically changed the ways by which one evaluates how to present a body of work. 21 original plates were printed on fine art, archival paper, and size 22x22 inches to meet more classic method of diffusion. The digital multimedia presentation was made available on CD and uploaded both on the author's website and YouTube, a highly visible Internet social media platform. The h.264 encoding of the final presentation also makes it available for viewing on IPods and mobile devices.

3. Conclusion

As it happens often in exploratory journey, I found challenges I was not expecting and got rewards I was not planning on finding. It helped me develop a deeper respect for the mathematic and scientific methodologies and the minds that brought us objective understanding of our physical environment. It also expanded my appreciation of the engineers and programmers that allow us today to expand the understanding of abstract visualizations in such complex terms. Computerized technology has made research, in particular in the field of iterated systems, much more dynamic.

The coherence of scientific and artistic concepts at the turn of the past century still stand today as solid technical and intellectual propositions all fields of communication can benefit from. The tools of mathematics, psychological and optical physiology enhance visual propositions in all forms of expression and expand its effectiveness.

Where do we go from there? [9]

This quick overview of concepts shared across disciplines by experts demonstrates again that, if our operating systems are neither finite nor perfect, the on-going collaboration of trained specialists in various disciplines continues to ensure that tomorrow will be brighter as it opens the door to new advancement in the understanding and appreciation of our circumstances.

Acknowledgments:

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