SUGGESTIONS FOR MAKING THE STUDY OF MATHEMATICS MORE ATTRACTIVE[1]

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Some of the texts with educational intent recently posted on ESMA may be confusing to some readers. Although they may also be of interest to a much broader public, these texts are intended primarily for students in primary and secondary form and content. It will go into some detail. Teachers who familiarize themselves with it may, in this way, be led to share their ideas with their students.

Data

Let us first briefly recall the goals that education seeks to achieve; no doubt there is a consensus on this matter. They include:

- training the mind,
- developing the capacity for judgment,
- acquiring knowledge,
- mastering skills.

1) What do we mean by the phrase, “training the mind”?

This refers to developing the aptitudes of sensibility, memory, observation, concentration, curiosity, imagination, and correct reasoning.

2) What do we mean by the phrase “developing the capacity for judgment?

This refers to the ability to express intelligent opinions in situations of incomplete information that, for the most part, will turn out to be essentially correct.

A) What is the role of mathematics in training the mind ? :

(i) Memory: Learning multiplication tables is still one of the best early methods for developing memory which, furthermore, is motivated by practical necessity.

1 I would like to express my deepest gratitude to Roy LISKER who completely rewrote my Franglish translation into a well written text, quite respectful of the original text (http://www.math-art.eu/Documents/pdfs/Suggestions.pdf)

(ii) Do schools still examine students, orally, or better in writing, on new ideas which teachers introduce them to, and which they are expected to understand?

(iii) Powers of observation : these are not encouraged by formal or axiomatic instruction, which moves directly to somewhat abstract notions. At the same time their physical or intellectual origins are ignored. The purpose of this is to create concise, rapid and concentrated presentations. However this is not a rewarding approach, either in terms of increasing understanding, nor in terms of the knowledge of the historical development of mathematics, and the relationship between mathematics with other intellectual activities. Proceeding in this manner in the name of mathematical efficiency is reductive; although it may be suitable to a small number of students, it is not successful for most of them. It is therefore indispensible that this way of introducing mathematical content must be revised. This will not hold back even the most talented students, as it forms connections with other subject matter.

(iv) Attention : Any formal presentation in mathematics that isn't immediately understood requires special consideration for the attainment of better understanding. This may also involve memorizing. To render this more effective one has to pay attention to the immediate presence of innate or induced psychological and environmental conditions. When these factors are not present, it is up to the teacher to try to increase the level of attention by returning to the original observations, by changing the conditions of observation, or through modifying certain properties of the objects under observation. Such methods can broaden the scope of knowledge, relax minds that tend to respond with resistance, or that become narrowed through contemplation on a single object.

(v) Curiosity and imagination : It is up to the teacher to draw attention to the unusual features of mathematical knowledge, to their level of generality, the analysis of the pre- conditions for existence, the ways by which ideas may be generalized, and their relationship with physics or more general knowledge.

It is by these means that the curiosity of students should be stimulated. Apparently it is rarely the case that teachers will address all these questions. Their time is taken up by administrative and other obligations and by the need to stick to the mathematical subject properly understood.

Attention to the meaning of mathematical concepts and facts can broaden the horizon of the student, even within mathematics.

Developing a many-sided view of a fact or concept awakens the imagination, and may also encourage students to enlarge their understanding, and conduct their own analyses in depth.

(vi) Reasoning ability: this enables one to explain things, ferret out the causes which, in their combination, explain the existence of facts and phenomena. The practice of deduction is present in every form of mathematics teaching. The difficulties that students find in presenting their demonstrations comes less from their ability to reason by inference, than from the difficulty they experience in uncovering the causes that allow one to make their deductions. A good knowledge of a subject makes it easier to foresee and deal with the simplest cases, if not all of them. From this point of view, exercises that strengthen reasoning ability are an important tool deepening mastery of any subject.

B) The place of mathematics in the formation of judgment:

One's judgment is shaped by one's culture. By culture we mean knowledge in depth of all the data, facts, hypotheses and theories in all domains of thought.

It is often the case in the modern world that first-rate researchers within a given discipline, for whatever reasons owing to lack of time, have a rather superficial culture when it comes to disciplines foreign to their own. At the same time, their success within their chosen discipline generally entails a deep understanding of the culture within that discipline. However, we cannot hope that a signal contribution to the problems of forming an adequate culture will come from the side of the specialists.

Mathematics, however, since it is relevant to every field of thought, can give one a window into every aspect of the world in which it has applications. Mathematical insights have a more or less universal relevance. Indeed they provide a realistic background for the right way to look at the world, one whose relevance is granted a priori.

Mathematics therefore must provide the basis for judgment. It should therefore be the task of mathematicians generally to promote this conceptual basis and its social relevance.

Education in mathematics, from the most basic to the most advanced, should form an organic whole. Any deficiency or lack of understanding at any place in the chain must affect the whole. In addition, physiological and psychological stress when combined with purely technical obstacles, can set up barriers in the mind of the student which prevents him from utilizing all the resources that a teacher can put at his disposal.

Findings

Looking only at the teachers in of contemporary primary schools, we see that, in the general diversity of their training, that mathematics is the poor relation. Even with those who've made the greatest progress in mathematics, their knowledge of mathematics is usually restricted to dependent on the subjects they learned at the university.

The French academic teaching of early years is heavily based on algebra (linear algebra, basics of group theory) and on analysis (up to solving partial differential equations). The proportionate amount of analytical and differential geometry is much lower. There is practically no differential topology. These are general trends, of course. Each specific instance must be analysed separately.

Both secondary and primary education are largely directed to preparing students to be ready for university courses. The relative amount of geometry is correspondingly poor. Basic arithmetic is taught in primary education. Most school teachers have a rather poor scientific training.

At the elementary and secondary levels one does finds training in the fundamentals. However, current French university practice tends to concentrate on knowledge alone, and far less of ability to use what one knows. The role of mathematics in culture is, for the most part, neglected altogether.

Goals

Our first aim is to "effect a reconciliation between the public and mathematics." The secondary objective is that of trying to alleviate the negative effects that accrue at various levels of education.

Proposals

1) Present an overview of the subject through mathematically inspired works of art, with guidance on the ways in which mathematics is employed to produce them. These works can be presented in exhibitions tailored to different kinds of spectators.

Films can be made of some of these presentations for a more general audience. There is a danger that such indirect viewing may be less valuable than direct exposure.

2) Relieve the inhibitions generated by academic formalism, through setting up a context that, to the public is more real and more agreeable. This can be done in both oral and written presentations.
3) The theatrical talent of the speaker can have a strong and lasting effect on his audience. This however, is usually in need of reinforcement. It is therefore desirable that the listener can listen a second time if so desired. Thus it is important that the lecture, lesson or presentation be captured on film.
4) Reading has definite advantages over watching a film: one is able to set one's own pace. This pace of reading is automatically established as a function of the time available to the reader and the degree of his understanding of the text. A reader can focus on a single word, look elsewhere for further information, and give himself time to digest whatever he is absorbing from the text. Reading provides more depth than the quick view of a parade of images; the eye much slow its pace as a function of the degree of undetrstanding present.

One should also therefore consider the possibility that short fictional narratives be written. By their nature these carry their readers into a more peaceful world, stimulating yet at the same time relaxing. They are not driven by the obligation to get through the exposition in an undeviating manner. The tale remains there, at one's fingertips; one can come back to it , one can ramble a bit, develop one's own commentary in peace. There are possibilities for fruitful exchanges, digressing then returning to the subject matter, and so forth.

Implementation

1) Overall Definition of Content :

This takes into account observations gathered in the paragraphs "Data" and " Observations ".

The main purpose of this is to acquire knowledge of the basics, either implicitly or explicitly, in all areas of mathematics. One aims to be as modern and timely as possible. As one moves forward in the curriculum the rate should be increased, especially on a technical level. Data and events from the physical world

– the true source of many mathematical concepts – should be emphasized. It is obvious that generalizations which are too abstract should be excluded.

Universal concepts highlighted by mathematics are present in all areas of thought and action: movement, stability, uniqueness.
(Related concepts : path, translation, rotation, deformation, probability, symmetry groups, bifurcation, unfolding, blowup.)

An universal tool : representation
(Related concepts : number, shadow, image, projection, map, representation.)

Topological and geometrical basic forms associated with fixed state or evolving objects are: open curves, closed curves (knots, circles), unfolding of knots, polygonal shapes as knots with singularities, balls and spheres, balls and spheres with singularities (polyhedral shapes), hollow and solid tori, cones.
(Related concepts : dimension, vectors and vector fields, fiber bundles and fiber-spaces, fibrations and foliations.)

2) Content :

The next objective is to present important facts related to the above lists of objects. Here are some suggestions. One can of course propose others:

One must first emphasize that the presentations are not directed to specialist or professional mathematicians, and have not all been designed by mathematicians. They are primarily intended for teachers. Save for very special cases, a totally rigorous approach is excluded.

The purpose of the presentation is to enhance the understanding of the significance of the ideas. Everything is presented in its historical setting.

Text labels http://www.math-art.eu refer to notions of a local character. The texts are mentioned here mainly for the images used to illustrate concepts; they should be considered simply written or spoken examples for students. They can be used training teachers who wish to convey and intuitive feeling for recent mathematics based on art. In this connection one can also recommend:

http://www.math-art.eu/Documents/pdfs/Catalogue_2013.pdf

http://www.math-art.eu/Documents/pdfs/ConferenceSaverne.pdf

http://www.math-art.eu/Exhibitions/expoIHP2010/pdfs/Catalogueexpo.pdf
Movement, translation, rotation :

A simple statement of the theorem of Aristotle-Liouville, that any local movement is composed of translations and rotations (angular translations). The composition of these two movements, the statement structure overall. Examples of trajectories :

ellipses and parabolas
http://www.math-art.eu/Documents/pdfs/glissades_Fevrier(bis).pdf,

epicycloid and trefoil knot
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf

Many teachers in today's world are teaching mathematics by offering a set of recipes. The mathematical background is virtually absent; this is both surprising and harmful. The following suggestions address this issue :

(1) The primary singular group: pion-pion exchange, rotations modulo π, elementary symmetries, reflection in a mirror.
(2) Conventional planar symmetries as examples of groups (translation, rotation, glide-reflection symmetry). Proof that the plane can be tiled with any quadrilateral. Conditions on vertex angles for general plane tilings.
http://www.math-art.eu/Documents/pdfs/patisserie/PM3.pdf
(3) Simple statement of the existence of 7 families friezes, 17 families of tilings of the plane. Examples of non-commutative rotations in space.
(4) Description of the hyperbolic disc
http://www.math-art.eu/Documents/pdfs/patisserie/PM3.pdf
Examples of tilings of the disk : it can be paved by any regular polygon
http://www.math-art.eu/Exhibitions/expoIHP2010/pdfs/Catalogueexpo.pdf
(5) Unfolding of the critical singular group. Two examples of the cyclic group (C_{12}, watch, and C_{10} rated by 10Z), eg subgroup C_4, the dihedral group of the triangle, of the square. Z as infinite unfolding of the singular group. Tiling of Z by 10Z, Z is the covering of 10Z (notion of covering). Description of the projection of the group Z on 10Z through the description of all elements of Z that project to a given element k of 10Z :

it will be called the fiber of Z at k, Z is considered a fiber bundle with basis 10Z. k is the shadow, the image of the projected fiber.

http://www.math-art.eu/Documents/pdfs/Kangourou.pdf

Representations:

Numbers and figures as representations of motion (translation; rotation (angle); translation followed by a rotation (Chuquet-Cardan number also named complex number)).

The different types of usual numbers (integer, rational, irrational, transcendental). Rationals can be represented by a finite sequence of numbers (proof given). Justification of the method used to show that the sum of two rational numbers is rational. Example: irrationality of the square root of 2, demonstrated. (Optional: the same for the square root of 3):

http://www.math-art.eu/Documents/pdfs/pythagore.pdf/cf

The shadow or image as representation: shadow of a vertical object illuminated by a "sun", a light source located at infinity; vertical rays, then inclined segments. The shadow, image, representation should not be confused with the object. Comparing shadows of two vertical segments: a statement of physical observation (known as Thales' theorem, static and dynamic versions):

http://www.math-art.eu/Documents/pdfs/patisserie/PM1.pdf/cf

Shadow or projection of an ordinary sphere on a plane, illuminated by a light source at infinity in the north-south axis, and at a finite distance from the north pole (demonstration). When the light source is at the north pole (stereographic projection) the shadow or image of a circle parallel to the equator is a circle (demonstration). The image of any circle not passing through the north pole is a circle (stated without proof):

http://www.math-art.eu/Documents/pdfs/patisserie/PM1.pdf

Statement without proof that this projection is conformal (preserves angles between curves drawn on the surface).

An other example of stereographic projection, the one of the two sheet hyperboloid (creation of the Creation Beltrami-Poincaré disk) :

http://www.math-art.eu/Documents/pdfs/patisserie/PM3.pdf

In this presentation, the number of elements of a figure projected onto the same point of the shadow is called the weight of the projection. It is almost everywhere constant on suitable levels of the figure (stated without proof) :

http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf

Basic forms:

The Point as the fundamental singular form. The scarcity of singular points: http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf

Unfolding of the trivial knot, i.e. a closed curve (the notion of dimension of a curve). Elementary deformations of a knot (ellipse, polygonal shapes), examples of non trivial knots:
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA23.pdf

In particular the trefoil torus knot :
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf

Singular points on a curve. Splitting a knot at a singular point, open curve. Trivial knots with singularities (polygonal shapes).

Pythagorean Theorem:
http://www.math-art.eu/Documents/pdfs/pythagore.pdf

Equations in Cartesian and polar coordinates of the circle.

Difference between geometry and topology - geometrical and topological dimension of a space, of a shape:
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA23.pdf

Rotation of a knot around an axis, in particular the equation of the rotation of of a circle around a diameter (the geometric 2-sphere).

Examples of deformation of the topological 2-sphere (ellispsoids, polyhedral shapes, the 5 Platonic polyhedra).

Counting the faces of an n-cube :
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf

The n-sphere as the boundary of the full n-sphere or n-ball.

The point and the knot as topological tori of respective dimension 0 and 1. Blowing up a point of a knot to another knot, simultaneously from all points of a knot : the hollow torus. Examples of geometric torus of dimension 2:

http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA1.pdf
http://www.math-art.eu/Documents/pdfs/patisserie/PM4.pdf

Angular representation of a torus. The torus as a fibered shape (or 'space'), whose base is the first knot, the fiber at a point being the knot resulting of the blowing up of this point. Cones as fibered spaces :
http://www.math-art.eu/Documents/pdfs/glissades_Fevrier(bis).pdf

Bands : examples of the cylindrical band, the Möbius band, its boundary is a trefoil knot. Concept of orientation:
http://www.math-art.eu/Documents/pdfs/bonneAnnee/BA23.pdf

Notion and examples of foliations :
http://www.math-art.eu/Documents/pdfs/patisserie/PM1.pdf

Fibrations as foliations. Properties of 'pastries', without proof. Concepts of plates, vectors, vector fields, baker's transformation:
http://www.math-art.eu/Documents/pdfs/patisserie/PM2.pdf