Art, Mathematics, Pedagogy

Claude Paul Bruter

Résumé. *One of the major activities of the mind is to represent. This activity establishes links between mathematics and art : only a few are briefly discussed in this article. They justify the presence of artistic works for cultural and educational purposes, addressing all audiences. We present a few examples taken from the activities of a European vocation institution dedicated to these purposes.*

1 Artworks and Mathematics as Representations

Maurice Ravel [1] used to claim not making any difference between the poet, the painter, the architect and the composer. He had grasped the universals of thought structure and thought functioning.

Valéry [2] was probably following this line of thinking when he expressed in his *Cahiers* : "Fortunately, I have established in 1892 the mathematical thinking as the standard for measuring the values of the mind on the one hand and, on the other hand, construction or specific experience ... I have since then only valued and esteemed an "artist" of any kind when his demands and (genuine) *freedom* were similar to the one of a geometer or a builder (e.g., of machines)".

Similarly, Henri Cartan, one of the fathers of the Bourbaki family, for the moment called in France the "Pope of Mathematics" declared : "In the speech I gave on February 1st, 1977 when receiving the Gold Medal of CNRS - The National Institute for Scientific Research in France - I argued that mathematics were more relevant to art than to philosophy". [3] Was he not implicitly expressing how the steps of thinking, no matter the field they applied to, are all part of the same unit ?

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This belief is based on a simple observation : the primary activity within our sensorial and nervous system leads to the representation of our environment, which is essential to ensure or spatio-temporal stability. Any representation, and ultimately any mental representation, can be externalized when creating artifacts imitating and symbolizing the environment.

But any representation is necessarily incomplete. A song with sorrowful or enchanting verses will convey the intense emotional and mental activity of a poet, but not his face features. No perfume rises from the melodious sound of a flute. The lips of a sculpted head are forever still. Painters and engravers will never make us hear the faint whispering of the breeze through leaves and foliage.

Fig.1 : Monet, Soleil couchant sur la Seine, effet d'hiver, 1880

All these representation are highly symbolic. Whereas it is rising or falling, the sun painted by Monet isn't the true sun. It rather symbolizes the presence of the sun and the effects of such presence on human senses in a specific context (see Fig. 1 & 2). Moreover, such artworks illustrate another characteristic of symbols : they are universal and spatio-temporally independent : artworks can be (theoretically) admired by the entire mankind ; across space and time, many men and women will indeed be exposed to them.

Finally, the drawing of the sun is characterized by the perfect form, the so-called disc of the mathematicians. In Monet's painting, obeying mathematical and pers-

Fig.2 : Monet, Impression soleil levant, 1872-1873

pective rules, a specific and major place was given to this disc.
In this specific aspect, the painter (who is primarily a draughtsman), composing his art, meets the geometer, the mathematician. Indeed, the latter also represents and symbolizes, and for this he primarily uses the drawing.

What are indeed numbers and letters, used by the geometer, the mathematician, if not originally drawings ? Across years, centuries and millennia, the form and semantic attached to such drawings have evolved, drifting away from concrete and immediate reality, gradually becoming symbolic elements.

The drawing ”1” shows primarily the presence of an object. The ”1” means that this object exists, with a huge potential of being embodied. However, each form of embodiment is secondary compared to the presence of the object. Such existence and presence are its deepest characteristics.

Fig.3 R. Mazoyer, The young Artist, 2006

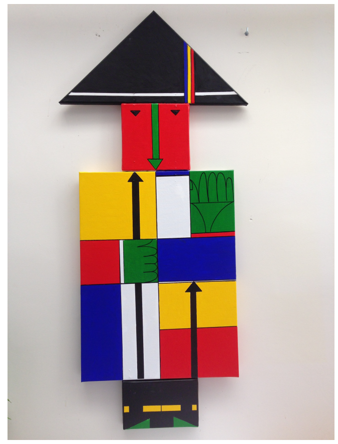

Fig.4 R. Mazoyer, the serious Artist, 2006

A representation is a portmanteau word, which agglomerates several components, including a source, such as a budding artist or an adult artist, and a support, such as the paper, canvas, computer screen or the material and intellectual tools allowing the representation to be created.

The transformation, which is everything made by the artist to create his artwork, which is a word that includes the act of performing the transformation, is represented by the mathematician by a single drawing, often that of the letter f. If C indicates the child or A the adult, the mathematician will note f(C) or f(A) their representation, their image on the canvas, that is to say the f-transform of C or of A.

The work of mathematicians seems to be similar to the one of other artists, except that their efforts are focused on the analysis and the study of the data arising from our environments. Such data are forms in a frozen state, the characteristics of their appearance and of their structure. They are the form of stationary objects, but also the forms of moving objects, of becoming objects.

A simple glance at the four paintings illustrating this chapter (Fig. 1-4) is enough to reveal what they have in common : an axis of symmetry. The axis is horizontal in the two first paintings ; vertical in the other two, and in all cases it structures the painting.

Symmetry is the manifestation of the balance of forces, which confers stability to an object or an artwork. Symmetry became a field of study in the late 19th century, following the lead of physicists, such as Pierre Curie. Understanding, recognizing its importance, and even its simple presence, took time.

The mere act of existing implies a balance between internal forces, and therefore the presence of symmetries. Thus, it is not surprising that the discovery and classification of ”elementary” particles have been largely relying on group theory, in which symmetry is a constitutive element.

These brief comments on four paintings and on the links between the thinking across different fields could belong to a larger study on the links between mathematics and arts. Some of these links are described for example in the Catalog 2013 [4] of the works from the ESMA funds, or in presentations made to last year the high school students in Saverne and Etampes [5].

2 Artworks serving Pedagogy

It appears that genuine artworks get important media coverage and have the power to attract large audience. This fact has been exploited by talented politicians. I shall quote for example these lines from the Ernst Kitzinger's very interesting book " Early Medieval Art " [6] :

"It was not, indeed, Charlemagne object to make artists appreciate and reproduce the outside word for its own sake. In fact, he and his contemporaries condemned art as a means of reproducing things which are obvious to the senses. They believed, however, in its usefulness in conjuring up the things of the past, and making them alive. Charlemagne had a very clear vision of the part that such realistic art could play in his work of political and cultural reconstruction. If art succeeded in giving physical reality to things which otherwise could only be grasped intellectually it could be turned into a powerful instrument of education."

This specific approach has recently been followed. Given that mathematics is impregnating all forms of art, appealing to the content of artworks for helping mathematicians to convey their message to a general audience became obvious. The ARPAM Association (Association pour la Réalisation du Parc d'Activités Mathématiques), and later on the European Society for Mathematics and Arts (ESMA, http ://www.math-art.eu), consisting of artists and mostly of academics, were established for this primary purpose. A brief presentation of the society and of its activities can be found in the recently published Bulletin 517 available online (http ://www.apmep.fr/IMG/pdf/Multimedia_no_517_Final.pdf) of the French Association of the National Education Mathematics Teachers (APMEP).

ESMA also organizes international seminars in Europe. The first one took place in Paris in 2010 (proceedings published by Springer [7]), the second one in Cagliari in 2013 (proceedings published by Cassini [8]) and the third one will be held in Ljubljana in 2016. The themes of these seminars were :

- *Theme* 1 : mathematical tools and software for the creation of artistic scientific visualizations ;
- *Theme* 2 : Analysis of artistic works from the mathematical point of view ;

- *Theme*3 : Pedagogical uses of scientific artworks.

One can be delighted to observe, in various disciplines, the increasing presence of educational experiences based on artistic tools. For example, the next International Congress on the Mathematics Teaching (Hamburg, on 2016) will hold a session entirely dedicated to mathematics pedagogy based on artistic works. Various realizations will be presented and ESMA will be present [9].

We predict that, in a few years, when these experiences and realizations will have multiplied and the analyses of such situations will me more advanced, the implementation of artworks in pedagogic set-up will become more widespread and better coordinated. However, one difficulty contributing to slow down such implementation could arise from some mistrust for the training of the teachers committed in this educational way.

As kindly and pertinently suggested by the reviewer, I shall here give an example of the use of some artwork to briefly introduce the succinct knowledge of a mathematical object. You shall first discover the notion of knot illustrated by two mathematical objects. You shall then see in which theories knots may appear. Finally you will once more meet a familiar concept from which one can illustrate a general behaviour of the mind. Please go to
http ://www.math-art.eu/Documents/pdfs/bonneAnnee/Bonne_Année.pdf
and reach the diapo 51 where you see this image :

Fig.5 : Jos Leys. "Anosov" trefoil

The golden ring is named a trefoil. If it is made with a piece of rope, you recognize a kind of knot as the one used in navy. From the end of the 18h century, mathematicians have developed a large theory, called the theory of knots. A knot is a curve in our usual space, like a thread whose ends are glued one to each other. The circle is a trivial knot. The trefoil is no more trivial, but very simple and elegant.

One other theory is the dynamical systems theory which studies the trajectories followed by moving objects like say an airplane. Now look at the animation from the following diapo 52 http ://www.josleys.com/gfx/DanseNDT_01.mov, you first see a point, a circle of null radius named here a singularity. It suddenly becomes a very very small circle, which in turn gets larger : you are attending a general phenomena called a blowing up. Then you see the circle which rotates in space around a diameter. The circle is lit say by the sun, and you see its shadow on the earth, on the horizontal plane. When the circle is in vertical plane, its shadow is a segment, a piece of straight line ! If you look at that segment, could you imagine that what you see is not the real object ? Now introduce the notion of deformation of a mathematical object : you can deform the circle made with a sewing cotton into a triangle, a kind of closed curve with three singularities. Then it rotates around its horizontal edge, and again, when it is in the vertical plane, its shadow is the same previous segment ! Well, do not take what you see as the reality, remind Plato !

Now the circle appears once more, and even two circles are appearing. These trivial knots are in symmetric positions. You cut them, then you deform the pieces and glue their ends so that you get two symmetric trefoils. And look : two people are dancing, and their respective trajectory is exactly a trefoil ! But these trefoils are symmetric with respect to a mirror located in the middle of them, so that you cannot stack them, they are different ! So there are two kinds of trefoils : unexpected, is not it ?

We predict that, in a few years, when these experiences and realizations will have multiplied and the analyses of such situations will me more advanced, the implementation of artworks in pedagogic set-up will become more widespread and better coordinated. However, one difficulty contributing to slow down such implementation could arise from some mistrust for the training of the teachers committed in this educational way.

The various contents of conferences, lectures, lessons, and practical exercises will give birth to reflections and probably to original work, both in the field of mathematics per se and of artistic creation. Note once more how the renewed alliance between mathematics and art were fruitful in the past, during the golden age of Renaissance with the introduction of perspective, the radiance of cubism in the first half of last century, or the creation of challenging decorative elements (knots, friezes, tilings) that in turn allowed the maintenance and dissemination of underlying mathematical tools.

Today, with the development of mathematics and of computer tools, old forms have been highly enriched, and newer have been created : more and more artists are seizing them. Naturally, museum curators are primarily working to support their rich artwork collections from the past and might be reluctant to integrate newer artworks (especially in old Europe), but such reservation should be overcome with time. We entered into a new era of human history, and contemporary artistic productions will reflect it, will represent it. In case galleries and museums are not accessible, the reader is invited to visit websites revealing to everyone all these novel and beautiful artworks :

http ://www.math-art.eu/exhibitions.php,
http ://www.imaginary.org,
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[2] Paul Valéry : Cahiers, tome 1, Gallimard, Paris, 1973, p.106.

[3] Henri Cartan : Le plaisir des mathématiques, R(evue) de M(athématiques) S(péciales), 115-1, 2004-2005, pp. 1-15.

[4] Bruter C.P. : Catalogue ESMA 2013, http ://www.mathart.eu/Documents/pdfs/Catalogue_2013.pdf.

[5] Bruter C.P. : Deux universels dans la décoration, premiére partie http ://www.math-art.eu/Documents/pdfs/Etampes/etampes.

[6] Kitzinger E. : Early Medieval Art, with illustrations from the British Museum and British Library collections, Indiana University Press, Bloomington, 1983.

[7] Bruter C.P. : (ed.) Mathematics and Modern Art, Springer, Heidelberg-Dordrecht-London-NewYork, 2012.

[8] Bruter C.P. : (ed.) Mathematics and Art III, Visualization and Diffusion of Mathematics, Cassini, Paris, 2015.

[9] Bruter C.P. : The work of art : an effective teaching tool, 13th International Congress on Mathematical Education, Hamburg, 24-31 July 2016. (urlhttp ://www.math-art.eu/Documents/pdfs/Bruter-ICME-13.pdf

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DOI 10.1007/978-3-319-46233-2_11

But all representations are necessarily incomplete. A song with sorrowful or enchanting verses will convey the intense emotional and mental activity of a poet, but not his face features. No perfume rises from the melodious sound of a flute. The lips of a sculpted head are forever still. Painters and engravers will never make us hear the faint whispering of the breeze through leaves and foliage.

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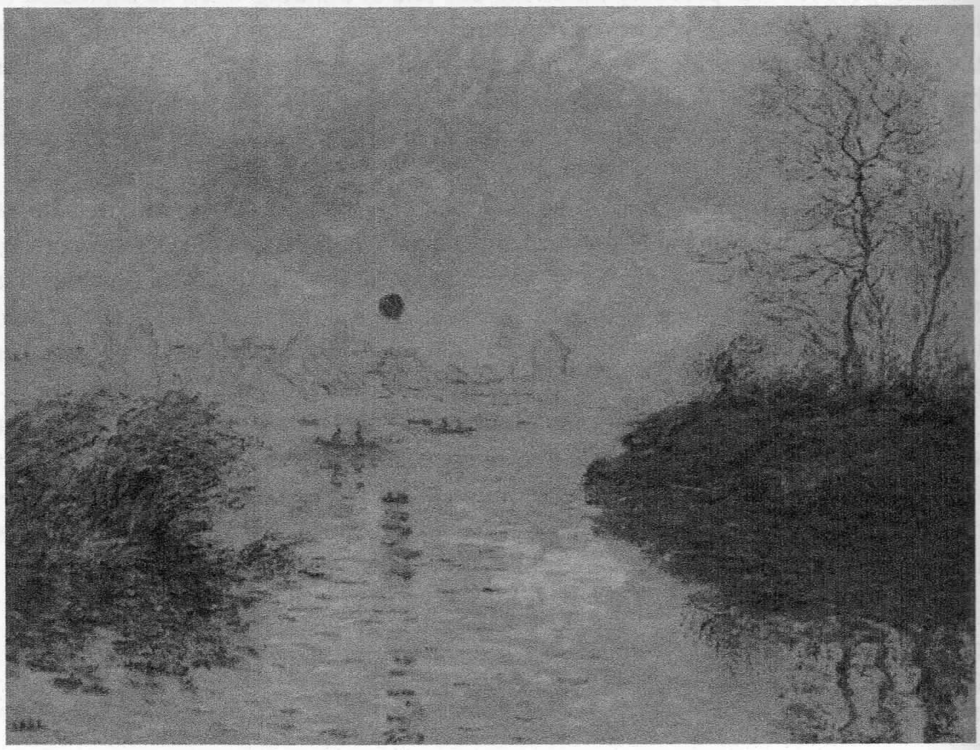

Fig. 1 Monet, Soleil couchant sur la Seine, effet d'hiver, 1880

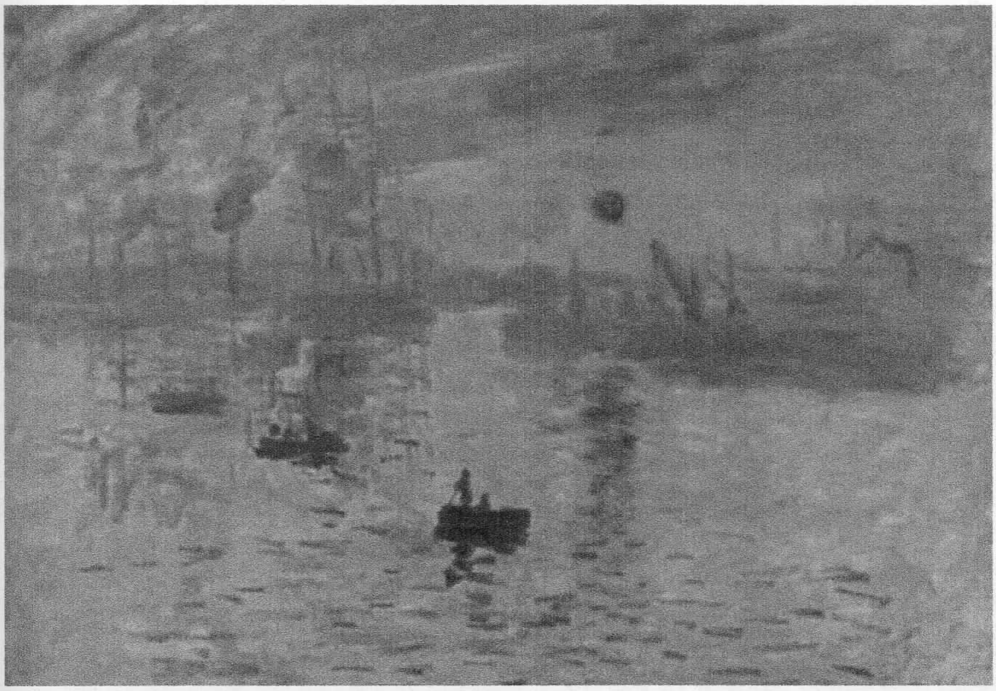

Fig. 2 Monet, Impression soleil levant, 1872–1873

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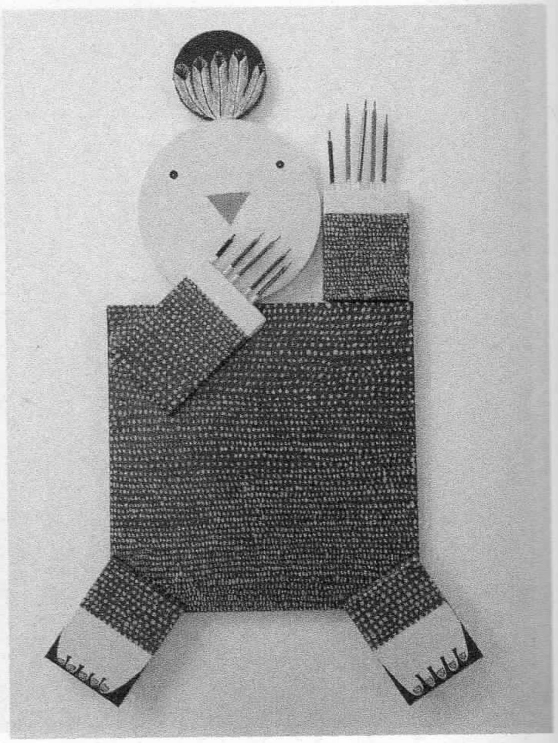
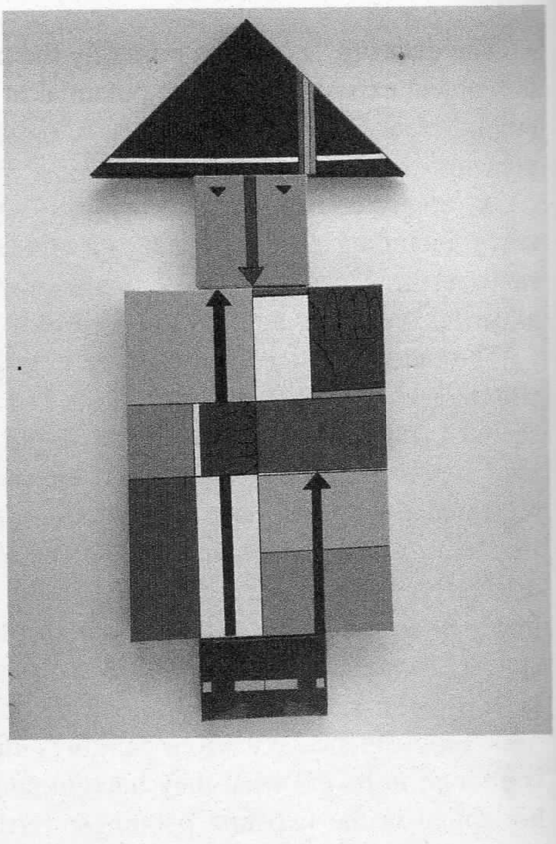


Fig. 3 R. Mazoyer, The young Artist, 2006. Artwork reprinted with permission



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As kindly and pertinently suggested by one reviewer, I shall here give an example of the use of artworks for introducing basic knowledge on mathematical objects. We will first discover the notion of knot illustrated by two mathematical objects and later on, see in which theories knots may appear. Finally, this example will also illustrate how our mind work and along the way, evokes some familiar concepts about reality and perception. Please go to http://www.math-art.eu/Documents/pdfs/bonneAnnee/Bonne_Année.pdf and reach slide 51 where you will see this image (Fig. 5).

Such golden ring is named a trefoil. When made with a line (to not call it a rope), you can recognize a kind of knot as the ones used by sailors. At the end of the 18th century, mathematicians started developing a vast theory called the knot theory. A knot is a curve in our usual space, like a wire with both its ends glued together. A circle is a trivial knot. A trefoil is not trivial, yet still very simple and elegant.

Another theory for which knots might be useful is the dynamical systems theory, which studies the trajectories of moving objects like, for example, airplanes. On the

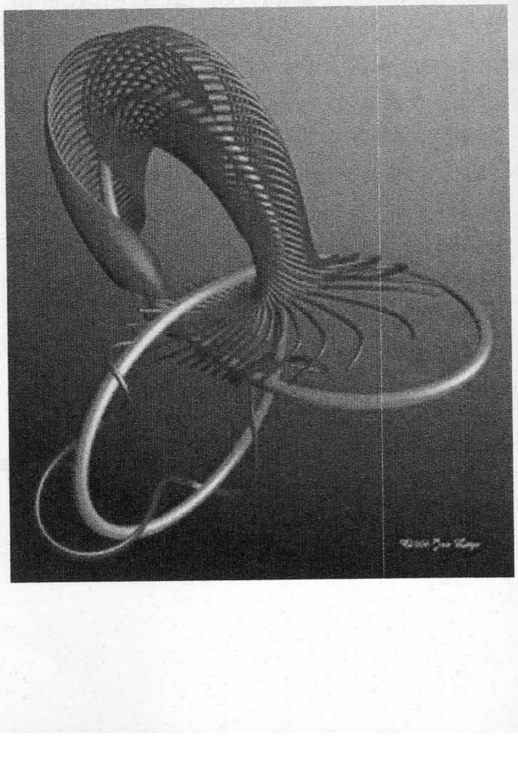

Fig. 5 Jos Leys. "Anosov" trefoil. Artwork reprinted with permission

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At the end of the animation, the circle is back. Actually, two circles are appearing. These trivial knots are in symmetric positions. They are cut, deformed and their ends are glued together: they are now two symmetric trefoils. And look again: two people are dancing, and their respective trajectories follow exactly the trefoils. These trefoils are symmetric, with respect to a mirror that would be located in between them, and they cannot be stacked together: they are different! Like for many biological molecules, there are two versions of the trefoil, perfectly symmetric yet different: surprising, is it not?

We predict that, in a few years, when these experiences and realizations will have multiplied and the analyses of such situations will be more advanced, the implementation of artworks in pedagogic set-up will become more widespread and better coordinated. However, one difficulty contributing to slow down such implementation could arise from insufficient training of the teachers committed to this educational way.

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