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THE WORK OF ART : AN EFFECTIVE TEACHING TOOL

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Very present on the sites of the AMS and the MAA, works of art constitute a very effective pedagogical tool. Some of these works and the demonstration of their utility will be treated in this article.

A FEW PEDAGOGICAL SUGGESTIONS

The diversity of teaching institutions, coupled with their internal diversity, local customs and traditions, as well as the diversity of their publics, lead us to tread cautiously concerning various suggestions and recommendations. This banal reminder of prudence and modesty can permit the reader to approach this text with the distance necessary to appreciate the inevitable insufficiences and the specific characteristics corresponding to his/her centers of interest.

The diffusion of a discipline at any level presents technical as well as affective aspects. There is often a tendency to overlook the latter for the former. The projects which I was able to put into place partially take into account the affective aspects. Before discussing these, it would be wise to call the reader's attention to the importance of the concurrent technical aspects needed to reach our objectives, listed as follows:

- to eliminate the psychological barriers separating certain auditors and publics from knowing about the mathematical world, its history and content.
- to introduce some basic elements to this knowledge.
- to instill in people the desire to further explore such knowledge.The objective is thus to practice a form of attractive initiation to mathematics, while bearing in mind that the content and methods must be adapted to the different publics to be addressed.

Concerning the acceptance or rejection of a discipline, the psychological factors inherent to each individual come into play. They are dependent not only on past and present environmental factors, but also on individual physiological capabilities, such as the ability to memorize or to understand, for example. Unfortunately, I don't know of any detailed studies of these phenomena, with which we might be able to better classify and better adapt the contents of the different exposés.

To resume, we should note the extended range of intellectual possibilities of those whose memory is iconographic, geographic, gestural, or auditory, sometimes linked to spontaneous outbursts of empathy, or on the contrary, withdrawal. We must also note the presence or absence of intellectual exercises favorizing the access to and the development of mental activities, such as reciting, memorization, calculating, and explaining, which help to maintain interest.

The connection between the content of a discipline and an individual's needs determine the acceptance or rejection of the said discipline. When both are in harmony, the individual can become addicted to his subject. She/he can become passionate about the subject at hand without necessarily being fully able to define its properties. Thus, the mental mechanisms, the evolution of the neurological process, may or may not spontaneously coincide with the internal mechanisms of mathematical structures, even though these make up the image of natural processes originating primarily from the physical world and deeply embedded in the constitution of all human beings.

The teacher must therefore endeavor to bring out the basic ties between the physical world and the elements constituting the mathematical universe, particularly via the fundamental phenomenon of stability, the latter being expressed in its repeatability and the universality of forms, images and structures. Addition, for example, ritualizes and symbolizes spontaneous and repeated natural acts, carried out in order to insure its temporal-spatial continuity. After this, it is logical to proceed directly to the idea of an additive structure. It then also becomes necessary to examine the objects in our environment which result from light effects. Practicing geometry is one way to develop a catalogue of the shapes of objects, to examine their properties and to follow their light effects.

We must first search the depths of our brain to show that the practice of mathematics is most natural, so that we can vaunt its merits. It can then become possible to create an emotional experience, coming from the unexpected, surprising our intellect and our senses. Where does the marvelous curiosity come from, which allows us to see that special straight lines issued from the apices of a triangle meet at a common point ? The teacher can awaken a joyous curiosity extending to other subjects, and ending up inexorably upon the stimulating and enriching desire to understand. The explanation can come later.

Rarity, novelty, originality of form and content, the exceptional qualities of its realization, these are the elements by which we qualify a work as beautiful or very beautiful. It strikes the mind, the senses, and marks the memory. For Emile Artin and his friend Henri Cartan, mathematics is an art, albeit an art reserved to a few mathematicians. The specialized technician and the true artist are still worlds apart. Art carries with it prestige, however, and to declare mathematics as an art gives it a certain aura and reputation before which we bow in deference, and which attracts as if it were a brand new bright star in the sky, an immense jewel burning with a thousand fires.

Before developing the formality and rationality gradually acquired through education, human beings have reacted promptly to their environment, whether fixed or moving. Early man approached his novel environment with curiosity and joy, discovering the unexpected, often charged with significance. The new objects and experiences were to be inscribed in his immediate memory. An external motivating explanation would contribute to fixing his attention and longterm memory.

Neither arithmetic, algebraic, analytical, or even category theories can be represented in a concrete manner. This can only be found in the fields of geometry and topology, for these fields begin by defining forms which can in large part be materialized or can become specific objects. The forms inherent to these objects acquire a substantial reality in the usual tri-dimensional space, by plays on light (in the figurative and literal senses) when placed inside well chosen environments which bring out their beauty. Sometimes we can move or change these objects, even touch or handle them. Of course, the longer the visual and physical contact remains, the deeper the features will be inscribed in memory and consciousness.

The teacher should first try to direct the listener's attention to the object's quantitative and qualitative specificities and to their relative disposition, pointing out the types of curves, holes, movement, deformations, or possible methods of construction. In the first phase, the goal is to gain the listener's confidence, both regarding the objects, now somewhat familiar, and regarding the warm and radiant person who is showing her/him previously unknown properties, now become interesting because of their universal character. Formalization, by introducing an adapted symbolism and causal manipulation of these symbols, can come later and even much later, when the first motivating information has matured and become acquired.

The interest in these objects and their explanation will be all the more strong depending on the object's esthetic qualities and the speaker's inspirational qualities. We should then try to choose among the very best artists, who understand how to render visible and accessible the objects emanating from the symbolic world of mathematics.

The teacher-presenter should discreetly point out the beauty of these creations. References to artists of the past supporting her/his exposé could serve as an opportunity to broaden the cultural horizon of her/his public. This has become all the more important, for despite the wealth of possibilities offered by the web, and because of the sheer weight of knowledge necessary to master a discipline, the very notion of a non-superficial general culture is tending to decline among the younger generation. The fine arts, sculpture and music represent our internal and external environment, just as mathematics does. As representational art, they share the same functional property in contributing to our temporal-spatial stability. They also present similar structural and universal characters. In showing the conscious or unconscious pervasion of mathematics in the realization of artistic works since time immemorial, we can point to an at times surprising mathematical presence within the modern works. Of course, this is also a way to introduce the works and the conceptual tools used in their creation, to awaken interest in humanity's past, to introduce knowledge, to open minds and broaden thought, in sum to arouse curiosity.

The interest in history goes much further. To cite certain major actors of the 19th century, such as Goethe or Darwin, "All true understanding is genealogical" (Darwin). I was happy to find this point of view strongly expressed by George Steiner, who wrote some wonderful passages concerning mathematics in his book entitled "Unwritten Books" (2007). I regret that the history of mathematics is almost always and at all levels absent from the mathematics curriculum. Particularly as this would strongly contribute to the interest level of the public, and at the same time give out basic knowledge of some past results, which have now become indispensable for the understanding of the present and future. I would, however, clarify what I mean by the history of a discipline. It is not merely a basic succession of dates and facts. It is also more profoundly about being able to reconstruct their origins, about how these ideas, processes, and inventions were born and took root in their authors' minds. While it seems obvious that detailed reconstructions would be nearly impossible to obtain, attempting to draw them along general lines could be a perilous, but very enriching exercise!

PROJECTS AND ACHIEVEMENTS

We can find chapters devoted to the pedagogical use of art works in each of the following books: *Mathematics and Art* [1], *Mathematics and Modern Art* [2], *Mathematics and Art III* [3], which can be consulted for more information. Concerning the reasoning, [4], [5], [6] can be consulted. Pedagogy-teaching methods will be on the agenda of the ESMA conference, to be held in Ljubljana in September, 2016. For more information, please check the ESMA site ESMA European Society for Mathematics and the Arts at www.math-art.eu or the organizer, mateja.budin@gmail.com.

Projects, such as a mathematics park and mathematics activities, expositions, exposés and their justifications, are all to be found on the ESMA site and in the bibliography.

Concerning the park and its visualization [7], [8], and [9] can be consulted. [10] concerns the expositions; [11], [12], [13] the exposés. The background for each story is given at the end of each text in [14].

References

[1] Mathematics and Art, Mathematical Visualization in Art and Education C.P. Bruter (Ed.), SpringerVerlag, 2002. ISBN 3-540-43422-4.

[2] Mathematics and modern art, Proceedings of the First ESMA Conference, Paris, July 19-22, 2010 C.P.Bruter (Ed.), Springer Verlag, 2012. ISBN 978-3-642-24496-4 e-ISBN 978-3-642-24497-1.

[3] Mathematics and Art III, Visual Art dans diffusion of Mathematics, Proceedings of the Second ESMA Conference, C.P. Bruter (Ed.), Cassini, 2015. ISBN 9 782842 251888

[4] Bruter, C. P. (2010). Fine arts to the service of Mathematics, EMS Newsletter 77, 19-20.

[5] Bruter, C. P. (2011). Les beaux-arts au service des mathématiques Gaz, Gazette des Mathématiciens 130, 83-90.

[6] Bruter, C. P. (2015). The pedagogical virtues of math and art exhibitions. In C.P. Bruter (Ed.) Mathematics and Art III, Visual Art dans diffusion of Mathematics, Proceedings of the Second ESMA conference (pp.1-7). Paris: Cassini.

[7] http://www.math-art.eu/Documents/pdfs/THE_ARPAM_PROJECT.pdf

[8] http://www.math-art.eu/Documents/pdfs/Bruter_Kozlov_Img.pdf

[9] http://www.math-art.eu/Documents/pdfs/ARPAM_Visualisation_Folies-002.pdf

[10] http://www.math-art.eu/exhibitions.php#1

[11] *Bonne Année* | Part I | Part II (NEW)

[12] *Pâtisserie Mathématique* | Part I | Part II | Part III | Part IV

[13] http://www.math-art.eu/Documents/pdfs/Etampes/etampes_2_reduit.pdf

[14] http://www.math-art.eu/tales.php