COLOSSAL CARDBOARD CONSTRUCTIONS

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Abstract

Impressive, large geometric sculptures can be made very cost effectively from cardboard by a group of people working together to cut the components and assemble them. In the process, the participants informally learn ideas about geometry and symmetry while developing team problem-solving skills and seeing first-hand how mathematics can be applied to art and design. Several examples are presented of "sculpture barn raisings" of this type that I have designed and led.

1 Introduction

I enjoy creating geometric sculpture both to challenge myself as an artist and because I feel mathematical art can have a pedagogical value for the viewer [1]. Carefully observing an unusual structure can put the observer into a mathematical mode of thinking - asking questions about the patterns and relationships inherent in the artwork. I have found that having a group of people help me assemble a geometric sculpture has an even stronger effect. Since the 1990's, I have been organizing events I call "sculpture barn raisings" in which I design a sculpture, fabricate the components from wood, metal, or other materials, and invite a community to participate in its construction [2][3][4][5][6]. Participants in these events get a hands-on introduction to the fun and creative sides of mathematics.

As awareness of my mathematical sculpture barn raisings has spread, more people have inquired about having me lead an event at their site. However, it can be difficult to find sufficient funding for the cost of purchasing and shaping permanent materials such as wood or metal. So in the past year, I have been experimenting by developing a series of designs which are suitable for fabrication in cardboard, which is much less expensive than wood or metal. Although it is not as strong, I can design for its properties and it has turned out to be sufficiently sturdy. Some of these sculptures have lasted many months and I expect they can continue to last much longer if treated gently.

2 Example Constructions

The examples presented here are all roughly spherical forms with icosahedral symmetry, in which all the parts are identical. That is not in any way essential to the larger ideas of this paper; it is simply my personal style in designing these works. Others might make large cardboard constructions based on very different mathematical ideas. My first experiment in this direction was at the Bridges Towson conference in July, 2012. For this, I designed a 1-meter diameter structure made of thirty painted rectangular components, each folded on a diagonal and joined using slots. See Figure 1 and the video of its assembly [7].

Fig. 1 One-meter cardboard construction made at the Bridges Conference 2012

Although the parts are simply slotted rectangles, the coloring aspect adds a certain richness and it was so successful that I went on to plan larger, more complex designs. Figure 2 shows a two-meter diameter cardboard construction built at a workshop I led at Southern Connecticut State University in October, 2012. A video of the assembly is online [8].

If there are several hours for the workshop, the participants can cut out the cardboard parts themselves as shown in [8]. If there is only an hour or so, then the parts need to be cut ahead of time. For cutting many identical parts, it is efficient to make stacks of the cardboard sheets, join them with "sheet rock screws" or clamps, and cut an entire pile at once with a saw. The part template

Fig. 2 Two-meter cardboard construction assembled from sixty identical components

is first traced on the top sheet of the stack. A band saw is ideal for this. With student groups, I have used a table-top scroll saw, which is not as fast as a band saw but is easy to transport to the venue and very safe compared to other types of saws. A spiral blade makes it easy to cut in any direction, which simplifies the process for people who have little shop experience. Younger participants can not use a saw, so their parts could be cut by others, perhaps ahead of time.

During the construction events, I can give detailed step-by-step instructions on how to assemble the parts. But if there is time, I can let the participants try to puzzle it out first. I have done this with teachers, leaving them confused for the start of the session. I think this is good to help them sympathize with what it is like to not understand the next step, as some of their students may sometimes feel. I also hope everyone then enjoys the "Aha!" experience when they do begin to understand the structure.

At some point in the workshop, I take time to explain the symmetry aspects of the design. It is useful to see the 5-fold, 3-fold, and 2-fold symmetry axes and use them as landmarks for adding additional parts to a partially complete construction. The chirality issue is also important to discuss. If the parts could be assembled in either left-handed or right-handed form, it is essential that everyone in the group make the same choice. A fundamental skill in much of mathematics is to learn to see patterns and extend them. In these designs, some visualization is required when geometric patterns are sometimes rotated or upside down from the exemplar. All of the designs shown here can be assembled in a modular manner at first. Groups can work in parallel assembling sub-units that combine into a larger structure. But due to the complexity of pieces getting in the way of each other, the final steps often require that individual pieces be inserted one at a time.

When designing for cardboard, there are many material issues to consider. Cardboard is available at low cost, but has relatively low strength, so is not suitable for long thin components. Corrugated cardboard has a grain, like wood, so folds easily in one direction while being more resistant to folding in the orthogonal direction. Aligning the template with the grain in the best direction can add significant strength to a part. Cutting with knives is not recommended as it is too easy to slip and cause injury. A band saw or scroll saw is much safer because the sharp part remains in one place. Using cardboard which is white on one side and brown on the other adds some visual interest while only slightly increasing the cost. In the US, a variety of sizes and thicknesses of cardboard sheets can be ordered and delivered through uline.com. Cardboard parts can be connected together with slots, clips, tie wraps, tape, and/or glue. Figure 3 shows a construction made from corrugated plastic election signs, which are freely available on the day after election day. This material is like cardboard in many ways but is slightly tougher and I used both slots and cable ties for the connections. This construction took place at Albion College, Michigan, in 2012. A video of its assembly shows the details, including the technique for using sheet-rock screws to hold together the stacks for cutting [9].

Fig. 3 Corrugated plastic construction made from recycled election signs at Albion College

Before making anything on a large scale, it it usually valuable to make a maquette. For most of these cardboard designs, I first made a paper scale model roughly 25 cm in diameter. The process is very useful for gaining insight into the structure and working out an efficient assembly sequence. It no doubt leads to a more robust final cardboard construction. Figure 4 and the video [10] show the paper model of a design which I later made in cardboard, 1.5 meters in diameter, with a group of teachers from Math for America in New York City. The cardboard construction is shown in Figure 5 and a video [11].

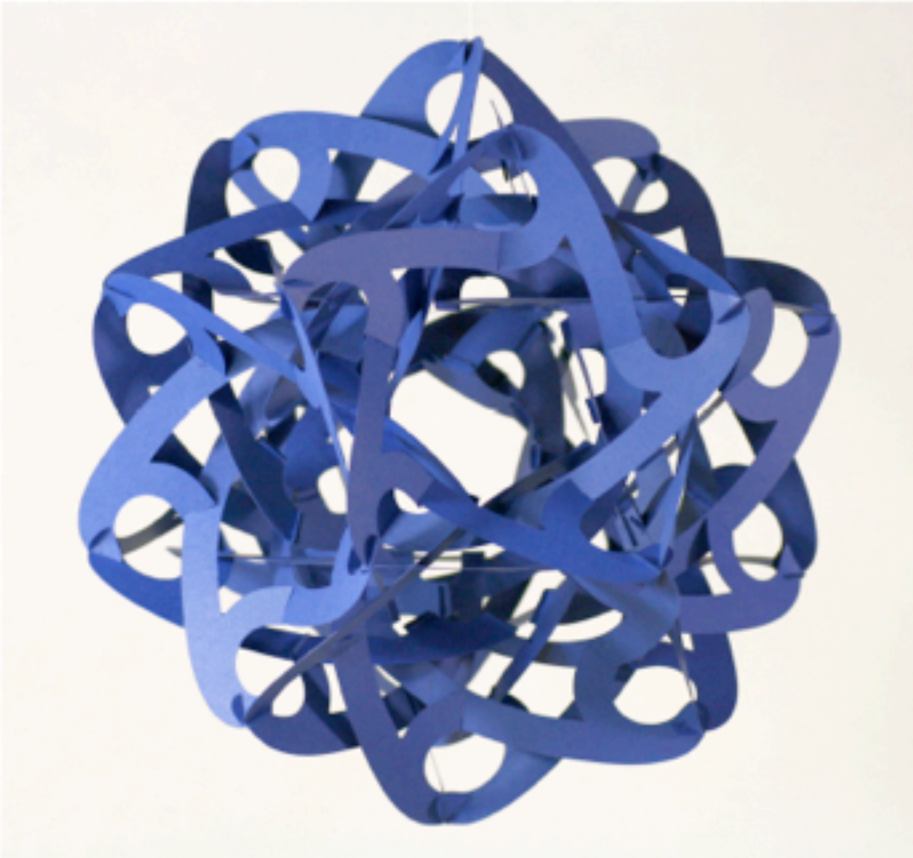


Fig. 4 30 cm paper model made in preparation for the large cardboard version of Fig. 5

Fig. 5 Cardboard construction made at Math for America teacher's workshop

After that, I worked with students in a workshop at Aalto University in Helsinki, Finland to make a larger version of the design from plywood sheets, shown in Figure 6 [12]. This suggests how any of these designs might be scaled up from paper to cardboard to larger dimensions using wood or metal, with corresponding changes to the connection system. In the paper version the parts are glued together; in the cardboard version the parts join with slots; in the plywood version cable ties are used.

After creating the design above, I continued to tweak it slightly in further versions. I changed the set of planes slightly so that the three pieces which meet at the exterior tips are orthogonal and surround an empty, small spherical space, as seen in the rendering of Figure 7.

Fig. 6 Plywood versions of Figs. 4 and 5 (left and right handed) at Aalto University

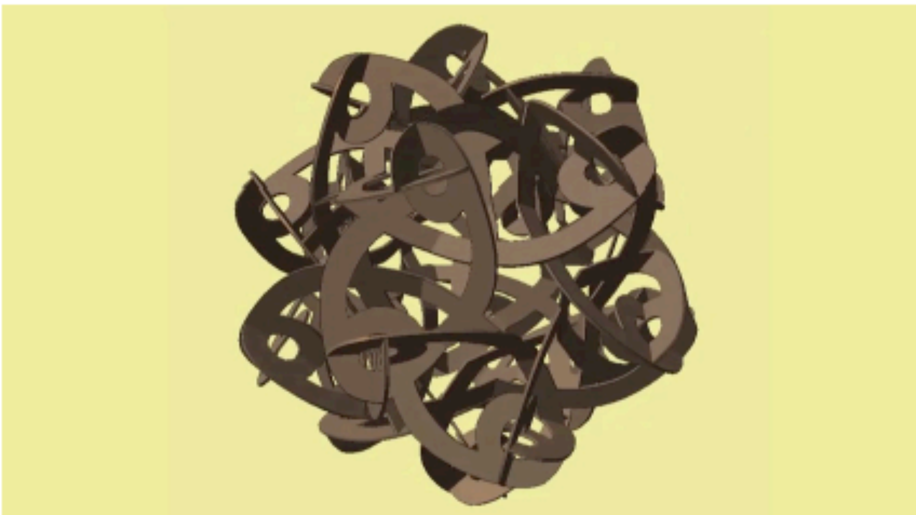


Fig. 7 Rendering before construction of design with orthogonal planes

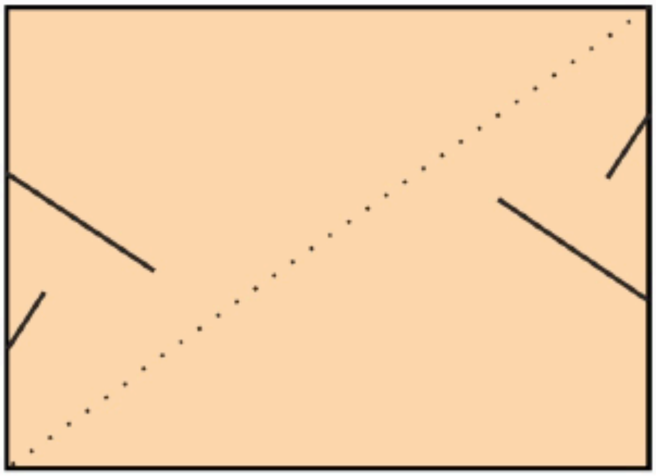
This is typical of how I see a computer model on the screen when designing and evaluating an idea, before committing to constructing it physically. I designed the examples shown here with the aid of a sculpture CAD program described elsewhere [13]. It understands how to arrange planes symmetrically in space and works with various symmetry groups. Working with it, I take into account material properties of cardboard and the connection system I envision.

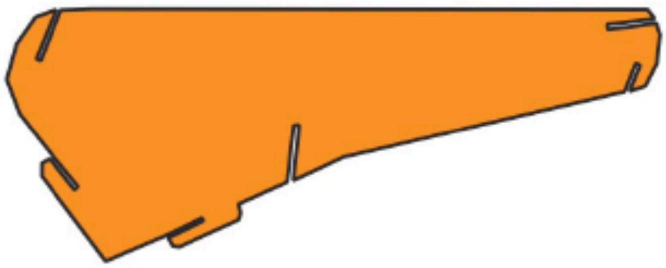
The two-meter cardboard construction of Figure 7 was built at a workshop I led at the Canada/USA MathCamp at Colby College, Waterville, Maine in July, 2013, shown in the video [14]. It is the first one where I tried using glue rather than slots. Brushing a thin layer of glue on folded flaps and holding them in place to dry with clamps takes longer than just sliding slots together, but it results in a stronger, permanent bond. So this is now my preferred method of joining cardboard. Participants must be cautioned not to use too much glue, or it takes much longer to dry. I then made one additional tweak for the version of Figure 8, built at St. Paul's School in Concord, NH, in November, 2013.

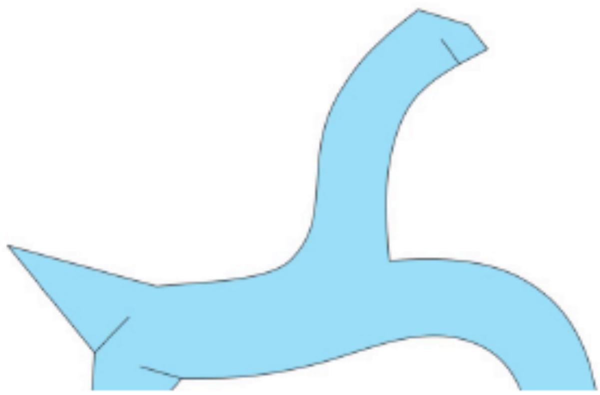
Fig. 8 Two-meter cardboard construction assembled from sixty identical components

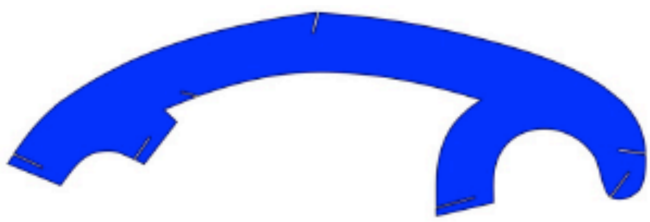
The change is not initially obvious in the completed structure as it only affects the interior. An inner opening which was triangular in Figure 7 was modified to be circular in Figure 8, in order to harmonize better with the circular exterior features. This change is more easily seen by comparing the last two templates of Figure 9.

Figure 9 provides templates for all the designs of this paper. Corresponding PDF files are available on my website [7]. To reproduce any of these constructions, I recommend printing the PDF at full scale and tracing it on to cardboard to make a master template which is then traced on to all the cardboard stacks to be cut. In some cases, marks must be made to indicate where parts are to join in the middle of a piece. I have found it is easy to mark these locations with small notches cut in to the edge of the template. When sawing, the notches are cut into the individual pieces and provide very clear alignment marks for guiding the assembly. Hopefully, the videos cited above give enough information for others to replicate the process.









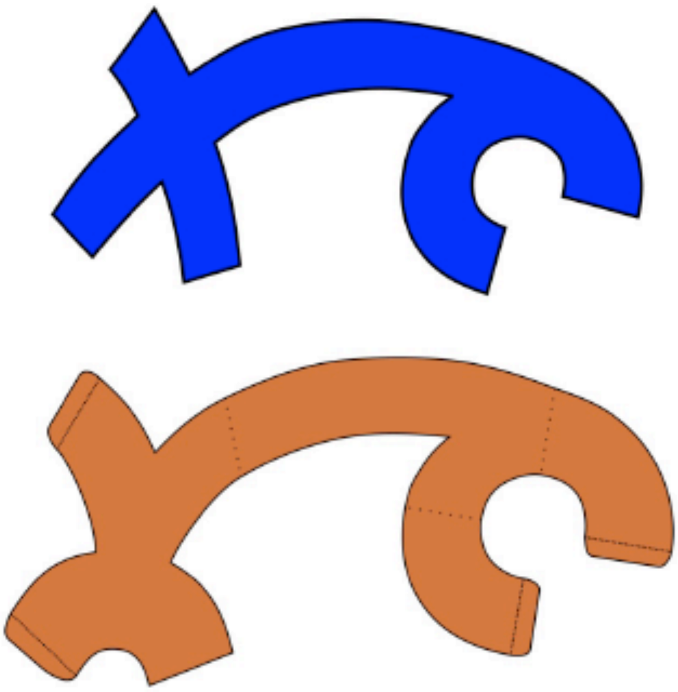


Fig. 9 Templates for the six designs above

3 Conclusions

I hope these constructions are not just visually engaging, but are replicated as fun challenging puzzles for developing collaborative problem-solving skills and a wider appreciation for the value of mathematics in art and design. While some artists are naturally possessive of their design ideas, viewing other's use of them as hurting their financial balance, I am happy to have others reproduce my designs if they want to make copies. Ugly corporate sculpture or shopping mall sculpture always depresses me when I see hundreds of people walk past without even a glance. So I find it personally gratifying when people not only look at my work but want to go to the effort of making their own copy. I always give permission freely and just ask that the copy be labeled "Design by George Hart" to distinguish it from the original instance where I was involved in the construction. It is great to see increasing interest and enthusiasm for mathematical art. I have additional designs on my drawing board and events scheduled, so I will be continuing to explore these colossal cardboard constructions in future work.

References

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