# MATHEMATICS AND ART III

# Visual Art and Diffusion of Mathematics

Proceedings of the Second ESMA Conference Cagliari, September 18-20 2013

Preface

The second Conference of the European Society for Mathematics and the Arts (ESMA) was held at the Department of Mathematics of the University of Cagliari (Italy-Sardegna) from 18 September to 20 September 2013. This volume gathers together the texts of the majority of talks held during the conference.

Since 2010, when the first conference was held in Paris, minds have been considerably and recently rapidly evolving. It seems now widely and quite accepted that the marriage of Art with Mathematics has a positive feedback on the intelligence and the acceptance of mathematics by most of the public.

When planing the conference, three main themes were defined:

- theme 1: Mathematical tools and software for the creation of artistic scientific visualizations
- theme 2 : Analysis of artistic works from the mathematical point of view
- theme 3: Pedagogical uses of scientific artistic works.

The main scope of the lectures was to give useful old or new tools to a wide audience to get in touch with some mathematics that artists could use to create new works. By focusing the attention on these tools, the idea was not only to enlarge the possibilities of creation by artists, but also to invite mathematicians to maybe locally deepen some of their theories.

An other aim was the promotion of math and art activities in the educational system, which are going on now.

There is only one contribution entirely focused on this third theme, emphazing the role of exhibitions associated with lectures, but most other contributions, concerning models, have some connection with it.

The historical article by Livia Giacardi mainly refers to the works of the Italian school at the end of the 19th century and the beginning of the 20th. At that time, algebra and analysis had not taken the important role they have got today, and

algebraic, analytical and differential geometry played an important place in the teaching at any level.

Some models and works are described and analyzed in the articles by most of the speakers.

The beautiful models in wood made by Joseph Caron during the same period are now currently exhibited in the library of the Poincaré Institute. They have been mathematically reconstructed by François Apéry. In some sense, Joseph Caron was a pionner of singularity theory. This kind of work may be understood as an encouragment to study generalizations of physical and mechanical devices to higher dimensions giving rise to richer mathematical configurations and illustrations.

A question remains without answer : for which courses did the Italian geometers and Caron use these models, and what was the pedagogical success of that uses ?

Note that the series of polyhedral models constructed by Richard Denner to illustrate the sphere eversion is typically in the spirit of all the previous models since they intend to be used as pedagogical tools. As George Hart did, he gives recepies to build the models.

But George'motivations are different from Richard's in the sense that his artistic passion was the main incentive to create his well known, beautiful, attractive and definitively original polyhedra. He gives the very detailed clues to reproduce them with a large size, and relates the various successful pedagogical experiences he made in the American context, where large groups of students gather to build up the artwork.

There is some similarity between that presentation and Dmitri Kozlov's one. Dmitri shows several people working at reconstructing one of his kinetic modules. These modules are cyclic periodic knots he first carefully studied as mathematical objects in a previous work. Materialized in metal or in fiberglass, being able to be constructed at any size, they can be used in architecture and for education as well. One of their characteristics is flexibility both in the material sense and in the

conceptual sense. They can be deformed, so that the same knot can move into the frame of a sphere, of a torus or of an hyperboloid. From the pedagogical point of view, these original modules share a new mechanical and mathematical interest.

Some analysis not only bring some new highlights on the works, but also allow to improve and extend them. That is for instance the case of Dough Dunham' study of Escher's work in 2-dimension hyperbolic geometry from which he constructs aesthetic triply periodic polyhedra. We are entering again the domain of real artistic works arising from the mathematical world.

Using various mathematical theories among which dynamical systems and time series analysis, the refined analysis of the scores of some well known composers by Renato Colucci and his colleagues allow them to propose algorithms which approximatively simulate the content and the structure of these scores. Then they can play to create interesting pieces of music which mix up different composers.

The work by Francesco De Comite is not focused on new mathematical research giving birth to new mathematical objects, but on a clever artistic use of former mathematics through recent computer tools he masters. He is a well known graphic artist, and shows a few of his original works coming from his favorite mathematics. Though he comes from computer science, one can say that, being a specialist of anamorphosis, he has a topological mind in the sense that he likes deformations, using the flexibility given by parameters. Bifurcation theory is related to creativity and life.

Artists or not, most people are today familiar with standard symmetry with respect to mirrors, or affine linear subspaces. There is no standard symmetry without underlying stability. On the other hand, standard symmetry is a special case of a more general symmetry with respect to any manifold, and which has to be seen first as local. When the manifold is a n-dimensional circle, the analytical formulation of that symmetry is called inversion. The article by Renzo Caddeo, Gregorio Franzoni and

Paola Piu use this inversion to fold a beautiful surface named the Dini surface, and to create a nice bouquet of such Dini surfaces. Nowadays, with the weakening of the teaching of geometry, inversion might be unfortunately ignored by many students. They will find here the possibility to get familiar with it, while artists could use it to get nice deformations of some of their objects and to improve their creations of works.

From the fact proved by Nash and Tognoli that any  $C^{\infty}$  manifold has a real algebraic model, polynomial representations are the most frequent to appear in the mathematical litterature devoted to geometry and topology. I would like to emphazise the usefulness of the Minkowski tricks used by Daniela Velichova to construct new shapes and which can be used to create numerical and visual representations of new mathematical objects that artists could use. I would like also to mention the fact that, apparently, there is no paper devoted to the mathematical study of this kind of representation.

I suspect that general cones defined by the last author could be better visualized through the previous means. I shall not comment anymore this theory, the introduction and the conclusion of the article are quite explicit.

During the Conference, we could visit a nice mathematical exhibition inside the «Citadelle dei Musei», and applaude a joyful play of theater written by our hosts. All the participants would like to thank Renzo Caddeo and his colleagues for the perfect organization of this Conference, in a warm atmosphere.

Claude Paul Bruter

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# Second ESMA Conference

September 18 – 20, 2013 University of Cagliari, Cagliari, Italy http://www.math-art.eu

## Conference venue

Aula Magna Matematica, Palazzo delle Scienze, via Ospedale 72, Cagliari

Exhibition venue

Cittadella dei Musei, Piazza Arsenale 8, 09124 Cagliari

### Main themes of the Conference

Mathematical tools and software for the creation of artistic scientific visualizations

Analysis of artistic works from the mathematical point of view

Pedagogical uses of scientific artistic works.

François Apéry (F)
Claude Bruter (F)
Francesco De Comite (F)
Renato Colucci (Co)
Vincenzo Iorfrida (I)
Dmitri Kozlov (R)

# **Invited Speakers**

Richard Denner (F)
Douglas Dunham (USA)
Michele Emmer (I)
Massimo Ferri (I)
Gregorio Franzoni (I)
Livia Giacardi (I)
George Hart (USA)

Patrice Jenner (F) Marcella G. Lorenzi (I) Andreas D. Matt (D) Francesca Mereu (E) Alexei Sossinsky (R) Daniela Velichova (SK)



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