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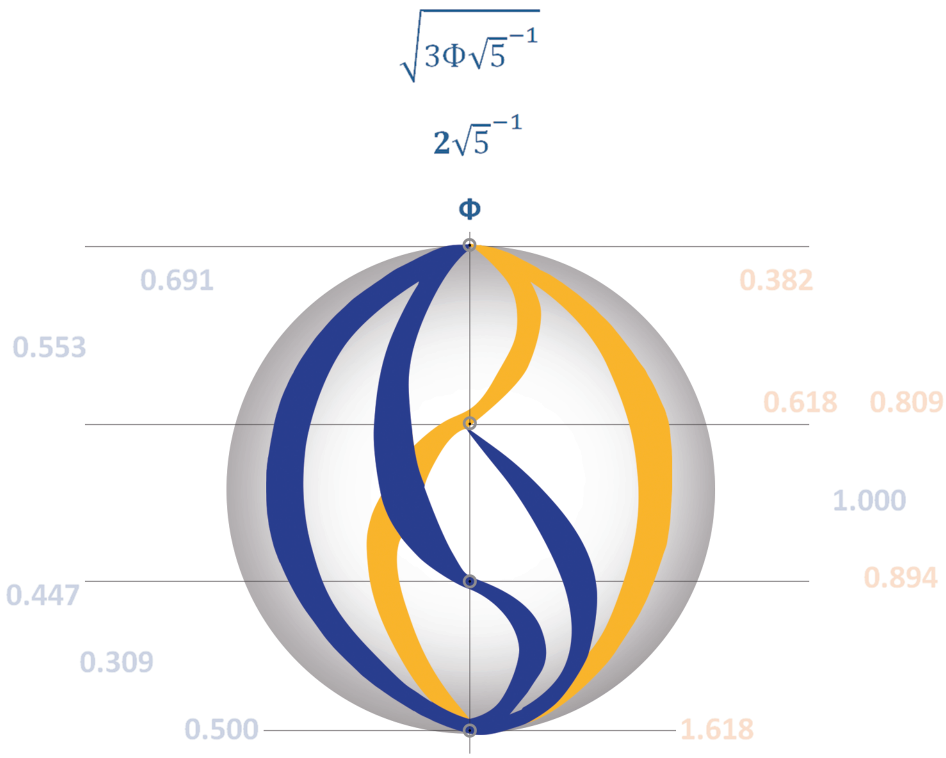
UNITS OF NATURAL GEOMETRY



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ABSTRACT

The problem of form-building in wildlife is the least developed field of knowledge about life and perception; a scientific discipline studying the unity and integrity of spatial forms is an outcast, because the concept "form" does not fit into any separate block of natural sciences, be it astrophysics, physics of microcosm, physiology, genetics or the art theory. It is all-inclusive and, consequently, "no one's". In architecture, sculpture, and painting, the face of harmony is fuzzy and amorphous. However, it is a strict science. It is a branch of geometry and the theory of numbers, in which the target of research is not geometric objects, figures, or numbers as such. They only add visual aspects to observations and findings. Harmony investigates the dimensional structure of things and living beings, the pace of divisions, and metamorphoses of bionic curves; it is a science dealing with the variety of forms of the real world, which all have a *single metaphysical source*. This book presents the general principles of generating forms and structures in wildlife and art. The reader will discover a special area of exact sciences, the way to which is found and determined by art, within natural sciences clarifying and nourishing it. It is *natural geometry*. Here, symmetry and the duality principle "contrari sunt complementa," dominating in nature and noticed by physicists and biologists, are generalized and integrated by the symmetry-of-pairs algorithm transforming the Pythagorean Theorem into the Golden Section. This insight into the old great truths has furnished the clue to modeling forms of wildlife and practical design. Harmony has started to speak the language of geometry and whole numbers, whose name "natural numbers" is not coincidental. The harmony language becomes understandable if the Unit (1), which represents in abstract terms real units-of-being, is seen as a structure: an elementary one, but already possessing the properties of symmetry and unlimited possibilities of combinatorics.

This is a summary of the author's semicentennial research. This work is divided into three parts. Part 1 is dedicated to algorithms of form-building and invariables of natural geometry. Part 2 models elementary forms of wildlife and space breakdown (combinatorics of golden polyhedrons). Part 3 defines the scale of proportions applicable in architecture and design and shows how eternal laws of harmony work in the art of architecture.

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Part 1

UNITS OF NATURAL GEOMETRY

1 Natural geometry is the key to the laws of harmony. There are three prerequisites to be observed in striving to pass from the geometry invented by the human reason to that adequately representing form-building in nature and being effective in creative work: 1) the world is structured, hence the number is a structure; 2) the world is twofold, which means that numbers, as well as the "point-sphere", are likewise twofold; 3) physical (energetic) interaction of infinitesimal elementary particles is subordinated to the principle "*Contrari sunt complementa*"[1] (*Opposites Are Complements*). In terms of numbers and geometry, the "opposite" will be understood as "**incommensurable**".

Α AND Ω

2 That which is extremely simple should *initially* bear in itself a source of complexity. Whence otherwise there would be brought about complexity of the real world**?** If it were granted that number "1" is out, then numerals 3, 7, etc. are free of sense. **There always are two numbers!** Any number presents a structure. But it is not enough to imply, it is necessary to **designate** the said structure. Let us denote integers by **α**. Also we shall assign them the second name – **ω**, which will reveal the structure of an integral number embodied into an equation.

$$\boldsymbol{\omega = \frac{\alpha}{1}}\textbf{, expression of Trinity} \qquad (1)$$

Such understanding of an integer possesses depth. It expresses *commensuration*, interrelation (–). It is a step to the universal unit – an abstraction depicting the metamorphoses of forms of the real world. **Existence of number $\boldsymbol{\omega^{+1}= \frac{\alpha}{1}}$ states existence of inverse number $\boldsymbol{\omega^{-1}= \frac{1}{\propto}}$.** Uniting inverse numbers, firstly, in two pairs, the difference $_{(-)}\boldsymbol{\omega} = \frac{\propto}{1} - \frac{1}{\propto}$ and the sum $_{(+)}\boldsymbol{\omega} = \frac{\propto}{1} + \frac{1}{\propto}$, and, secondly, in two pairs of pairs (2), implies the law of doubling and halving. If the pair of pairs is linked by subtraction, we see doubling of the inverse number $\frac{\mathbf{1}}{\propto}$; if the pair of pairs is linked by adding, we see doubling of the straight number $\frac{\alpha}{\mathbf{1}}$:

$$\left(\frac{\propto}{1} - \frac{1}{\propto}\right) - \left(\frac{\propto}{1} + \frac{1}{\propto}\right) = \mathbf{2} \propto^{\mathbf{-1}}\mathbf{;}\ \left(\frac{\propto}{1} - \frac{1}{\propto}\right) + \left(\frac{\propto}{1} + \frac{1}{\propto}\right) = \mathbf{2}\propto^{\mathbf{+1}}. \qquad (2)$$

However, this is not a trivial doubling ($\propto + \propto = 2\propto$). Differential binar $_{(-)}\boldsymbol{\omega}$ *falls short* of the source by an inverse number; summation binar $_{(+)}\omega$ *exceeds* the source by the same number! We are confronted with the algorithm of doubling and halving, preservation and modification taken together, the unique and the only mechanism of metamorphoses in biology, *replication*, the creative tool of searching for *new* structures, adaptation of living systems to current changes.

THE SECOND PYTHAGOREAN THEOREM (SPT) AND THE GOLDEN SECTION

3 Two spheres can be combined into one without loss of their individuality. Let us draw two circles *AB* nested into each other, one of which is formed by points W, the second by points V. Triangles *AWB* and *AVB* possess a common hypotenuse, and there are infinitely many triangles. Duality recreates itself, uniting all that is becoming into something whole. 1) From *two squares* there emerged *one*; from *one two*. 2) The doubling converts a square into a double square; bisecting the square parallel to a side generates two double squares; 3) the second cut divides

[1] As formulated by Niels Bohr.

the double square *diagonally* into two rectangular triangles, opening the door to the Golden Section two times. Firstly, by commensurating side 2 with the diagonal increased by minor side 1, and secondly, by commensurating the same side with the diagonal reduced by minor side 1.

The double Pythagorean Theorem makes it possible to express **Trinity** with a single symbol. It is unity, simultaneously a number and a visual image, a sphere. Generic points of two spheres W and V nested into each other are two poles, *A* and *B* (Fig. 2.1,2). Any other point of sphere V cannot coincide with any point of sphere W. Spheres W and V are interleaved, they interpenetrate each other. Thus two spheres make up one sphere, a "sphere-the-third", a whole (Fig. 1.1,5). Condition: legs of triangles W (segments **A, B)** and legs of triangles V (segments **a, b**) are **incommensurable** – the key to algorithm Φ, the code of self-reproduction of Life, the law "from one two, from two one," "from one all things and from all things one."

$\Phi^{+1} = (\sqrt{5}+1):2 = 2:(\sqrt{5}-1) = 1.6180339...$ $\qquad$ $\Phi^{-1} = (\sqrt{5}-1):2 = 2:(\sqrt{5}+1) = 0.6180339...$

4 Unit **ω** is presented in the likeness of a sphere, where the distance between poles – segment *AB* – is variable. When the approaching extremities of the diameter, poles *A and B*, are superimposed, we have a Point. There is one *point, but at the same time there are two points.* We have presented this with the points of sphere W on the left half of the drawing and the points of sphere V on the right one. As there are two spheres, the Pythagorean Theorem became ***doubled***. Connection of points $\mathbf{W_n}$ with poles ***A****, B* (a set of paired numbers A, B) is described by equation $A^2 + B^2 = \mathbf{c}^2$. Connection of points $\mathbf{V_n}$ with poles ***A****, B* (a set of paired numbers a, b, with incomparable numbers A, B) is described by equation $\mathbf{c}^2 = a^2 + b^2$. The Pythagorean equation has doubled and found a symmetric form, reminding a soaring bird with two straightened wings:

$$A^2 + B^2 = c^2 = a^2 + b^2. \qquad (3)$$

Let's transfer number $\mathbf{a}^2$ of equation $A^2 + B^2 = a^2 + b^2$ from the right side to the left, and number $\mathbf{B^2}$ – from the left to right (we interchange their positions). This permutation ($\mathbf{a^2} \rightleftarrows \mathbf{B^2}$), has transformed the doubled (Second) Pythagorean Theorem to a four-letter code, from this point on – "*the symmetry-of-pairs equation*":

$$A^2 - a^2 = b^2 - B^2 = (A + a) \times (A - a) = (b + B) \times (b - B), \text{ whence}$$

$$\frac{A + a}{b + B} = \mathbf{N} = \frac{b - B}{A - a} \qquad (4)$$

In the unique case, when **N=Φ**, the symmetry-of-pairs equation becomes infinitely combinatorial and meets all of the requirements set forth in Paragraph 1. The doubling (numbers 1 and 2) and right angle have created (by the Pythagorean Theorem) the diagonal of double square equal to $\sqrt{5}$. Identification of the doubled Pythagorean Theorem with number **Φ** (the Golden section) occurs when number $\mathbf{W_n}$ is supplemented with sphere $\mathbf{V_n}$ up to the "sphere-whole"**,** which is filled in by **numbers whole to base** $\sqrt{5}$. The alloy of duality and fivefold symmetry is created by condition $\mathbf{a} = \alpha\sqrt{5}$, $\mathbf{b} = \beta\sqrt{5}$.

$$\boldsymbol{\omega} = \frac{A+\alpha\sqrt{5}}{\beta\sqrt{5}+B} = \boldsymbol{\Phi} = \frac{\beta\sqrt{5}-B}{A-\alpha\sqrt{5}}. \qquad (5)$$

Here $\Phi^{+1} = \left[\frac{\alpha\sqrt{5}+A}{B+\beta\sqrt{5}} = \frac{B-\beta\sqrt{5}}{\alpha\sqrt{5}-A}\right] = \left[\frac{\gamma\sqrt{5}+C}{D+\delta\sqrt{5}} = \frac{D-\delta\sqrt{5}}{\gamma\sqrt{5}-C}\right]$ = etc.

Permutation of numbers $\mathbf{a^2} \rightleftarrows \mathbf{B^2}$ radically alters the *Pythagorean* equation. Before the permutation, this is geometry where the vertices of the right angles, and points **W** and **V,** create a spherical surface. After the permutation, this is the Symmetry-of-Pairs equation (3), the symbol of an energy event.

5 This expresses the essence of sphere rather than its form. Now the equation does not describe the addition of catheti in points **W** and **V**. Instead, it describes interaction of two forces concentrated in two poles A,B, i.e. genetically identical but opposite points. The set of number pairs packed at pole A (A $\pm$ α $\sqrt{\mathbf{5}}$) is put in correspondence with the set ($\beta\sqrt{5} \pm$ B) packed at pole B. All of the pairs are in a state of stable (golden) dynamic equilibrium:

$(A + \sqrt{5}) : (\beta\sqrt{5} + B) = \Phi.$

This is possible under the following condition: interrelations A $\leftrightarrows$ B and a $\leftrightarrows$ b is prohibited; interaction of pairs A, $\alpha\sqrt{5} \leftrightarrows$ B, $\sqrt{5}$ is allowed. Abstract representation of infinite set of double spheres W, V (the Second Pythagorean Theorem) is backed up with interaction of two infinitely powerful potentialities concentrated instantly and inexplicably in poles A and B.

*There emerged a metaphysical image of the Creative force which is simultaneously present everywhere. Thus unit **ω** = **Φ** (Fig. 2.1) was established as **the first invariable of natural geometry**.*

$$\Phi^{+1} = \tfrac{1}{2}(\sqrt{5}+1) = \mathbf{1{,}6180339....};\ \Phi^{-1} = \tfrac{1}{2}(\sqrt{5}-1) = \mathbf{0{,}6180339...}$$

Any number of natural sequence can play the role of numbers A, B, ***α, β***. But only the occurrence of fivefold symmetry gave the algorithm the role of the form-building law of nature. Numbers incorporate in pairs; the pairs combine into pairs of pairs (from one two, from two one) in a unique pattern: the rule of doubling-and-dichotomies *shapes the structure as a whole, and its details as well*. In equation (5) everyone of numbers (A, ***α***) in the numerator is built *of halves* of numbers (***β***, B) in the denominator; everyone of numbers (***β***, B) in the denominator is built *of halves* of numbers (A, ***α***) in the numerator. [2]

$$\boldsymbol{B}\ \left\{ \begin{array}{ll} \boldsymbol{\alpha} = {}^{1}/_{2}\,\beta + {}^{1}/_{2}B; & \boldsymbol{\beta} = {}^{1}/_{2}\,A - {}^{1}/_{2}\,\boldsymbol{\alpha}; \\ \boldsymbol{A} = {}^{1}/_{2}\,\mathbf{5}\,\boldsymbol{\beta} + {}^{1}/_{2}B & \boldsymbol{B} = {}^{1}/_{2}\,\mathbf{5}\,\boldsymbol{\alpha} - {}^{1}/_{2}\,A \end{array} \right\}\ \boldsymbol{A} \qquad (6)$$

The halved units, when combining in pairs, give rise to life *of two new Units*.

$$1 = +\frac{\Phi}{\mathbf{1}} - \frac{\mathbf{1}}{\Phi};\ \sqrt{\mathbf{5}} = +\frac{\Phi}{\mathbf{1}} + \frac{\mathbf{1}}{\Phi}.\ \Phi^{+1} = +\frac{1}{2}\mathbf{1} + \frac{1}{2}\sqrt{\mathbf{5}};\ \Phi^{-1} = -\frac{1}{2}\mathbf{1} + \frac{1}{2}\sqrt{\mathbf{5}} \qquad (7)$$

There has arisen a unique ring, in which *reasons are consequences of consequences, and consequences are reasons of reasons:*

$$\mathbf{\Phi = f\,(1,\sqrt{5})\,;\ 1 = f\,(\Phi);\ \sqrt{5} = f\,(\Phi)} \qquad (8)$$

A NEW INSIGHT INTO THE FIBONACCI-LUCAS SERIES

6 It has become possible to decipher structures (2) and (9), which is of key importance in natural geometry. As said above, additivity enriches the natural geometry with the *replication*

[2] As this takes place, a rule is observed: both numbers of numerator should be either even or odd. The same rule shall be observed in denominator.

algorithm. Multiplicativity makes it possible to present a higher level Unit. It outlines a rhythm of changes, a ring of interrelations shaping a whole, Unit ω. Pairs of pairs: the differences and sums of inverse numbers constitute doubled pairs of pairs; they combine in chains of four elements, which consistently are *multiplied by themselves*. There came about a chain in which exponent n of each element naturally grows in the next link from n = 0 to n = 1, n = 2, n = 3, etc.; $n \rightarrow \infty$.

$$_{(-)}\omega_n = [\frac{\Phi}{\mathbf{1}}]^n - [\frac{\mathbf{1}}{\Phi}]^n \ ; \ _{(+)}\omega_n = [\frac{\Phi}{\mathbf{1}}]^n + [\frac{\mathbf{1}}{\Phi}]^n \qquad (9)$$

Table 1. Algorithm of reproducing biostructures

SERIES L (LUCAS, module 1) and SERIES F (FIBONACCI series, module $\sqrt{5}$)
CONSTITUTED THE "DOUBLE HELIX" while EMBEDDING THEMSELVES INTO EACH OTHER

Exponent n	$\propto^n$	Left branch Difference $_{(-)}$ ω $\left[\frac{\propto}{1}\right]^n - \left[\frac{1}{\propto}\right]^n$			Right branch Sum $_{(+)}$ ω $\left[\frac{\propto}{1}\right]^n + \left[\frac{1}{\propto}\right]^n$
0	Φ^0= 1.000000	$\left[\frac{\propto}{1}\right]^0 - \left[\frac{1}{\propto}\right]^0$ =0	θ 0	N 2	$\left[\frac{\propto}{1}\right]^0 + \left[\frac{1}{\propto}\right]^0$ = 2.000000
1	Φ^1= 1.618034	$\left[\frac{\propto}{1}\right]^1 - \left[\frac{1}{\propto}\right]^1$ = 1.000000	N 1	θ 1	$\left[\frac{\propto}{1}\right]^1 + \left[\frac{1}{\propto}\right]^1$ =2.236068
2	Φ^2= 2.618034	$\left[\frac{\propto}{1}\right]^2 - \left[\frac{1}{\propto}\right]^2$ = 2.236068	θ 1	N 3	$\left[\frac{\propto}{1}\right]^2 + \left[\frac{1}{\propto}\right]^2$ = 3.000000
3	Φ^3= 4.236068	$\left[\frac{\propto}{1}\right]^3 - \left[\frac{1}{\propto}\right]^3$ =4.000000	N 4	θ 2	$\left[\frac{\propto}{1}\right]^3 + \left[\frac{1}{\propto}\right]^3$ =4.472136
4	Φ^4= 6.854102	$\left[\frac{\propto}{1}\right]^4 - \left[\frac{1}{\propto}\right]^4$ = 6.708204	θ 3	N 7	$\left[\frac{\propto}{1}\right]^4 + \left[\frac{1}{\propto}\right]^4$ = 7.000000
5	Φ^5= 11.09017	$\left[\frac{\propto}{1}\right]^5 - \left[\frac{1}{\propto}\right]^5$ = 11.00000	N 11	θ 5	$\left[\frac{\propto}{1}\right]^5 + \left[\frac{1}{\propto}\right]^5$ =11.180339
6	Φ^6= 17.94427	$\left[\frac{\propto}{1}\right]^6 - \left[\frac{1}{\propto}\right]^6$ = 17.88854	θ 8	N 18	$\left[\frac{\propto}{1}\right]^6 + \left[\frac{1}{\propto}\right]^6$ = 18.000000
	Φ^7=29.034443	$\left[\frac{\propto}{1}\right]^7 - \left[\frac{1}{\propto}\right]^7$ = 29.00000	N 29	θ 13	$\left[\frac{\propto}{1}\right]^7 + \left[\frac{1}{\propto}\right]^7$ =29.068 883

etc.

The even right and odd left "units" of this sequence make L-branch of this structure. It is an additive series of natural sequence. The first numbers of this series are 2 and 1. So the series appears as follows:

2, 1, 3, 4, 7, 11, 18, 29, 47, etc. These numbers belong to the Lucas series, well-known in biology just in the form we are used to see it.

The second branch, complementary to that of Lucas, is composed of even left and odd right numbers of the same sequence. It is a F-branch, the additive Fibonacci series. But it does not belong to the "natural numbers" series. Mantissas of its constituent numbers are infinite decimal fractions:

0, 2.2360..., 2.2360..., 4.4721.., 6.7802.., 11.1803.., 17.8885.., 46.9574..., etc.

And nevertheless, these numbers are *whole* ones, but whole to module $\sqrt{5}$ and *incommensurable* with number 1***,*** as demanded by the complementarity principle!

Thus the idea of inverse numbers (trinity) shows that there are not two series illustrating the life reproduction mechanism, but a single bifurcated series. Two branches of the combined Fibonacci-Lucas series are interleaved, nested into each other. Two "parallel lines" of this series are twirled in a double "golden helix". Numbers whole to module 1 and those whole to module $\sqrt{5}$ are connected pairwise. Each link ("helix turn") represents a complementary-opposite pair. Fundamental biological structures are arranged in the same fashion.

It is common knowledge that the ratio of contiguous Fibonacci series numbers (as well of those in Lucas series) tends to **Φ**. But the Lucas and Fibonacci numbers making a whole $_{(-,+)}\omega_n = [\frac{\Phi}{1}]^n \mp [\frac{1}{\Phi}]^n$ are the *golden numbers with absolute accuracy.* It is not only a limit of Lucas and Fibonacci series as commonly cited.

The beauty of this double algorithm is amazing, affinity of its structure to that of a DNA molecule, which is responsible for keeping similarity of hereditary units to those of an initial prototype, – catches the eye. Is it not a metaphysical sense of the Golden Section? And whether is it possible, strictly following mathematical logic, to extract number Φ from the idea of integrity which merges the law of spatial isolation of units-of-being and the unity of parts and whole for each of Units by the harmony law involving the algorithms of structurization?

INTEGRITY

7 Let's assume that there is something stand-alone: number ω. Boundlessly copying itself and being multiplied by itself, it aggregates everything that is formed in this process in a universal whole, referred to as number **1**.

$$\sum_{n=1}^{\infty} \omega^{(+n)} = 1, \quad \sum_{n=1}^{\infty} \omega^{(-n)} = 1 \qquad (10)$$

It is just algorithm of Integrity: life and movement. The structure of number 1 is open. The basis and root of number 1 are halving and doubling; numbers ***ω*** are equal to **1/2** and **2/1.**

If $\sum_{n=1}^{\infty} \omega^{(+n)} = 1$ then ω = **1/2,** (11.1)
If $\sum_{n=1}^{\infty} \omega^{(-n)} = 1$ then ω = **2/1.** (11.2)

When having generated divarication, equation (10) splits. There emerge two its branches: equation (12) and equation (13). Their coming to being may symbolize breaking of the World down to the world of crystals and that of live organisms.

The chain of numbers **ω** with *even places* in equation (10) gave rise to an equation, which root is number $\sqrt{2}^{\pm 1}$ (inorganic world):

If $\sum_{n=1}^{\infty} \omega^{(+2n)} = 1$ then $\omega = \frac{1}{\sqrt{2}}$, (12.1)
if $\sum_{n=1}^{\infty} \omega^{(-2n)} = 1$ then $\omega = \frac{\sqrt{2}}{1}$. (12.2).

The chain of numbers with *odd places* in equation (6) gave rise to an equation, which root is the Golden Section number **Φ** invariably present in structures, rhythms and forms of live nature:

If $\sum_{n=1}^{\infty} \boldsymbol{\omega}^{+(2n-1)} = \mathbf{1}$ then $\boldsymbol{\omega} = \Phi^{(-1)}$, (13.1)

If $\sum_{n=1}^{\infty} \boldsymbol{\omega}^{-(2n-1)} = \mathbf{1}$ then $\boldsymbol{\omega} = \Phi^{(+1)}$. (13.2)

Amazing capacity and completeness of metamorphoses of number $\boldsymbol{\omega}$ stem from the initial property of binar Φ, which is *trinity*.

VISUAL IMAGE OF UNIT Ω AND THE SECOND INVARIABLE

Sphere

8 A sphere may be conceived as a Point, a closed "space-atom", a planet, the sun, a live cell nucleus, expansion of the Universe. *The sphere* bears on itself the rules of form building used by the nature. Let's imagine a binary sphere with axis AB. On drawing, it is a circle. Everything relevant to sphere W shall be drawn to the left of vertical axis AB and everything relevant to sphere V – to the right. Sphere W presents ratios of positive integers (whole to module 1), sphere V – ratios of numbers whole to module θ =$\sqrt{5}$.

Let's continue dichotomizing.

Let's divide left semicircle AB in a point W_0 into *two* parts so that segments W_0A and W_0B were connected by doubling: W_0A = A = *1,* W_0B = *2.* According to the Pythagorean Theorem, $1^2 + 2^2 = (\sqrt{5})^2$, diameter $AB = \sqrt{5}$. From similarity of triangles AW_0B and φrB it is obvious that the distance from center φ to segment AB is equal to *half* of initial segment W_0A, $r\varphi$ = 1/2. Cathetus W_0B = 2 *is divided in two halves by point* **r**. A chain of dichotomies and its important consequences are plain to see.

1) The emergence of point **r** makes tangent W_0B capable to inscribe sphere *ab = 1* into sphere $AB = \sqrt{5}$; as a result, the number of spheres has doubled *(Fig. 2.1).*

2) The dichotomy of cathetus W_0B effected by point *r* (W_0B: 2 = 1) produced the advent of point W_1 and, thereby, led to the *trichotomy* of cathetus W_1A (*Fig.* 2.2): circle *ab* = 1 splitted segment AW_1 in points r_0 and r_1 *into three equal parts,* each of them equal to number $\sqrt{2}^{-1}$.

$$W_1B = r_1A = r_0r_1 = W_1r_0 = \sqrt{\mathbf{2}}^{-1}$$

Point W_1 created connection between numbers 1-2-$\sqrt{2}$ -3-$\sqrt{5}$, in trinity $W_1B : W_1A$ = **1:3**.

3) *The number of spheres has tripled*. Three dichotomies nested three spheres into each other. Their diameters are interrelated as numbers

$$AB : ab : mn = \sqrt{5} : 1 : (\sqrt{2})^{-1}. \qquad (14)$$

Central core of this threefold structure is sphere ***mn*** = $2^{-1/2}$. Number $\sqrt{2}$ plays the major role in the world of inorganic natural forms (crystals) and in art. An unlimited set of spheres is embedded in sphere AB since points of sphere W,V are linked to the poles with unlimited set of ratios. We can mentally return all of them into the Point of origin, so as to present circle AB = $\sqrt{5}$ as vanishingly small something – a point – and as expanding Universe ($0 \le AB \to \infty$).

The sphere contains all conceivable alternatives of the symmetry-of-pairs algorithm execution. Going from a structure to structure, from a link to link graphically represents movements of segment WV that joins complementary points W and V on the binary sphere.

Their co-ordinated movement opens two infinite sets of numbers: integers N and their complementary numbers (incommensurable to 1), which are integers of the second kind (we name them numbers θ). In general, it is an image of expansion (Fig. 3.2-3). Here each pair-of-pairs combined of integers N is answered with a pair-of-pairs combined of numbers θ whole to an irrational module, and each numerical pair-of-pairs is answered with a corresponding sphere. Sphere Ω presents an image of movement: the space-time continuum contracted into the Point of origin.

The second invariable of natural geometry

9 Growth of integers N and θ, metamorphoses of geometric objects – all that is explicitly presented on a plane by movement of segment $\mathbf{W_nV_n}$, which *in-motion* dissects a circle in a ratio of the Golden Section. End points W and V of segment WV slide along circle AB. If point W moves to the left of pole A to pole B, V moves, on the contrary, to the right of pole B to pole A. Points W, V do not approach each other and do not move apart; such is the scape of the starry sky. Distance WV remains unaltered in relation to diameter AB:

$$\mathbf{W_1V_1} = 2ab = \mathbf{2/\sqrt{5}\,\mathit{AB}} = 0.8944272\,AB. \quad (12)$$

That is *the second invariable of natural geometry* (*Fig.* 3, 2,3).

Let's present the Second invariable as a 3D image. Segment WV turns around a sphere with diameter ab = 1 (on plain drawings, spheres are mapped as circles). Each new position of segment WV changes the angle of its intersection with axis AB, at the same time changing numerical image of the Golden Section. Ever new symmetry-of-pairs equations (SPE) arise; SPEs formed from pairs of paired integers accumulate numbers of ever major magnitude (see Supplement, Table 3).

Each new symmetry-of-pairs equation appears in 3D representation as ***three pairs*** of conic pyramids built by five moving segments. Two segments are the catheti specified by natural integers (N=1); two others are the catheti specified by numbers whole to module θ = $\sqrt{5}$. The fifth segment (invariable WV = 2/$\sqrt{\mathbf{5}}$ AB connects apexes of right angles $\mathbf{W}_n$ and $\mathbf{V}_n$ (*Figs.* 3 and 4). Just one operation, rotation of this closed structure about axis AB through an angle of 2π, inscribes in a sphere two "UFOs", big and small ones, coming in touch at point "k", which is the general apex of two cones and a cross point of the Ptolemy tetragon diagonals. The big "saucer" encloses sphere N = ***ab = 1.*** The sphere is inscribed in a cone built by turning of invariable **WV** about axis of this sphere (Fig. 4).

10 There are symmetry-of-pairs equations unsuitable for inscribing sphere ***ab*** in sphere AB with the use of *second constant* WV. It needs to clarify this supervenience.

The equation converting *the Pythagorean Theorem to the Golden Section* has a left and right sides. Each side has a numerator and a denominator. Metamorphosis – transformation of the left side into the right one – proceeds as follows: both the numerator and denominator interchange their positions, and also the signs connecting numbers change to their opposites. Connection of complementary numbers in pairs can be expressed by equations of the form ($\frac{+}{+} = \frac{-}{-}$) or ($\frac{+}{-} = \frac{+}{-}$). In the first case ($\frac{+}{+} = \frac{-}{-}$), the left side of equation is created by addition, as is demanded by the Pythagorean Theorem. Contrastingly, the right side is a mirror-antisymmetric reflection of the left.

$$\Phi = \frac{A+\propto\sqrt{5}}{\beta\sqrt{5}+B} = \frac{\beta\sqrt{5}-B}{A-\propto\sqrt{5}}. \qquad (5a)$$

It is a correct algorithm. The surface of sphere (points W,V) is given by the Pythagorean Theorem: the parts are *added* in a whole (+).

In the second case ($\frac{+}{-} = \frac{+}{-}$), we observe a diverse picture. It seems to be logical and consecutive in the context of binarity and symmetry. But the law "Opposites are complementary" is here interpreted in a different way. Signs within each side of the equation, in numerator (**+**) and in denominator (**–**), *are opposite*. Whereas signs in the left and right sides of equation (numerator contra numerator, and denominator contra denominator) have turned from the opposite to *the identical*:

$$\Phi = \frac{A+\propto\sqrt{5}}{\beta\sqrt{5}-B} = \frac{\beta\sqrt{5}+B}{A-\propto\sqrt{5}}. \qquad (5.b)$$

Graphical presentation of equation ($\frac{+}{-} = \frac{+}{-}$) revealed a contingency: segment WV ≠ $2/\sqrt{\mathbf{5}}$ AB. It fell short of being an invariable. Relation SPE = Φ is preserved, but segment WV does not reproduce sphere ab = 1 (Fig. 6.1-4, SPE 6-11). Testing by a rule: "Each of numerator numbers (A,α) is compiled from *halves* of denominator numbers (β,B); each of denominator numbers (β,B) is compiled from *halves* of numerator numbers (A,α)" leads to a paradox. Positive numbers β turn out to be negative, negative numbers appear as positive (Fig. 6,5; Supplement, Table 4):

in	SPE-16	we get	β = +17 = -17
	SPE-17		β = -1 = +1
	SPE-18		β = -3 = +3, α = +13 = -13,
	SPE-19		β = +67 = -67, etc.

Point V representing a number of type **θ** (β) has a counterpart, point **V'**. A number belonging to set **θ** appeared in territory of numbers **N;** it is number β mirror symmetric with respect to axis AB (Fig. 6.1-4). It has reproduced sphere ***ab*** = **1** out of tetragon AWBV built by the Pythagorean Theorem. In other space. **WV' =** $2/\sqrt{5}$ AB**.** It is pertinent to put *a sphere produced by an imaginary constant* into correspondence with the *imaginary* ***Unit***. Let's assume that ***ab'*** **=** $\sqrt{-\mathbf{1}}$.

THE THIRD INVARIABLE OF NATURAL GEOMETRY

11 The Golden Section is the first invariable of natural geometry. The first and second invariables are mutually dependent. By action of its movement, ***the second*** invariable, segment WV = **2**/$\sqrt{5}$ AB, created core **ab = 1** nested within ***sphere*** AB**,** which action decomposed the whole (AB) with relation to Φ. Dividing of sphere AB in two spheres W and V, plus emersion of sphere **ab**, have converted a trivial sphere to the "golden sphere". Four triads of the Golden Section came into being:

Ab : ba = ba : aA; Ab : ba = ab : bB; Ba : ab = ba : bB; Ba : ab = ba : aA (Figs. 2.1, 9.2).

Combinatorics is a powerful tool for reaching the main objective of the nature. It is a way of searches for structures and forms favorable for survival. The doubling-halving method is a key to combinatorics. Number Φ ≡ symmetry-of-pairs algorithm has no rivals in solution of this

The second invariable connects points W_0 and V_0; it combines doubling of unit 1 and the reciprocal of $\sqrt{5}$. ***WV = 2***$\times \sqrt{\mathbf{5}}^{-1} AB$ = 0.8944272 AB. Point W_0 is connected to poles A and B by distances in the ratio 1:2.

The idea *of binarity* assumes the second spatial separation of numbers N and θ. Numbers N and θ can be separated so that they were arranged not in one orbit (AB), but in two different orbits, AB and ab. We shall transfer points W (the pairs of numbers N) onto sphere ab, leaving points V (the pairs of numbers θ) on sphere AB (Fig. 7-8). Alternatively, we can transfer onto sphere ab the pairs of numbers θ and points V (now they are points $\boldsymbol{v}$), while having left points N on sphere AB, and then connect complementary pairs of points $W_n \boldsymbol{v}_n$. Alternative choices (what points to move, and what to leave on sphere AB) do not change the outcome. An important point is that numbers N have been separated from numbers θ, and in both cases distance **W**$\boldsymbol{v}$ retains *the same constant value*. Having connected points **W** and $\boldsymbol{v}$ (Figs. 8.1,2 and 9.2,3), we have discovered *the third invariable of natural geometry,* segment **W**$\boldsymbol{v}$**.** What is the role of the third invariable?

12 *The third invariable*[3] $W_0 v_0$ implies, firstly, *tripling of number Φ (3Φ), secondly, appearance of a reciprocal to* $\sqrt{\mathbf{5}}$ and, thirdly, *embedding of number 5 into the square root of square root,* ($\sqrt{\sqrt{\ }}$).[4]

$$\mathbf{W_0 v_0} = \sqrt{\mathbf{3\Phi \times \sqrt{5}^{-1}}}\ \textbf{ab = (1.4733704...) ab}; \qquad (16)$$

$$\mathbf{W_0 v_0} = \sqrt{3\Phi \times \left(5\sqrt{5}\right)^{-1}}\ AB = 0.658911...AB$$

A sphere with radius $\tau\omega$ = $\boldsymbol{\tau'\omega}\sqrt{\frac{1}{\sqrt{3\Phi \times \sqrt{5}}}}$ = 0.303531 … is inscribed into sphere ab = 1.

Number 5 is allowed under the root-of-root sign; it is a way to the depth without bottom. Each step here encounters a riddle without an univalent answer because extraction of a root is a reverse of multiplication. It is a mystery, since (+) x (+) = +; (–) x (–) = +. And here we see one more mathematical fact deserving focused attention.

The principle of doubling and halving has sequentially, step by step, nested into a sphere with diameter $AB = \sqrt{\mathbf{5}}$ three more spheres put into each other:

a sphere with diameter ab = 1,

a sphere with diameter mn = $\sqrt{2}^{-1}$

a sphere with diameter $\boldsymbol{\tau\omega} = 2 \times \sqrt{\left(3\Phi \times \sqrt{5}\right)^{-1}}$.

---

[3] Figs. 9.3 and Fig. 6.2 $\mathbf{W_0 v_0}$ **= ?** ka = $\Phi^{-1} - \sqrt{5}^{-1}$. kv_0= ka + 1. $W_0 v_0 = \sqrt{kW_0{}^2 + kv_0{}^2} = \sqrt{\frac{3\Phi}{\sqrt{5}}} = \sqrt{2{,}1708204}$ = 1,4733704...

[4] **Proof** (see Fig. 2-3). From similarity between $\mathbf{W_0}$k and $\varphi\boldsymbol{\omega}'\boldsymbol{v}_{\mathbf{0}_0}$ follows: $\varphi\boldsymbol{\omega}'$ = $\sqrt{\left(\mathbf{3\Phi \times \sqrt{5}}\right)^{-1}}$ **=** 0.3035310..

$\mathbf{W_0 v_0}$: $\boldsymbol{\tau'\omega}$ **=** 1.4733704 : 0.3035310= 3Φ.

1.4733704 × $\sqrt{5}$ = 3.2945564... = 0.3035310^{-1}.

Invariable $\mathbf{W_0 v_0}$ ***(the end of event)*** is equal to the core radius increased by a factor of 3Φ (***the origin of event "becoming"***).

Sphere $\boldsymbol{\tau\omega}$ is a core of structure Φ. It is delimited by translation of the third invariable $\mathbf{W}\boldsymbol{v}$ = 1.4733704 along circles AB = $\sqrt{\mathbf{5}}$ and ***ab* = *1.*** The core radius is $\boldsymbol{\varphi\omega}$ = $\left(\sqrt{3\Phi\times\sqrt{5}}\right)^{-1}$ = 0.3035310.

Connection between invariable $\mathbf{W}\boldsymbol{v}$ and core diameter $\boldsymbol{\tau\omega}$ embedded in the Φ-sphere center by movement of this invariable is of fundamental importance.

The concept of number in natural geometry means inseparable existence of straight and inverse numbers: The existence of number $\boldsymbol{\omega}^{+1}$= $\frac{\alpha}{1}$ signifies the existence of inverse number $\boldsymbol{\omega}^{-1}$= $\frac{1}{\alpha}$ ". The being of both numbers *coincides in time* (see Section 2). At the same time the third invariable responsible for nesting the core, $\mathbf{W}_0\boldsymbol{v_0}$ = $\sqrt{3\Phi\times\sqrt{5}^{-1}}$ = 1.4733704, and radius of this core $\varphi\omega' = \sqrt{\mathbf{3\Phi\times\sqrt{5}}}^{\,-1}$ = 0.303531 are connected ***through time interval*** **θ** = $\sqrt{\mathbf{5}}$ (!!) as inverse numbers. It is a mathematical fact: having increased the third invariable by a factor of $\sqrt{\mathbf{5}}$, we find ***a number reciprocal to core radius*** $\varphi\omega'$**.** [5]

3.2945564... = 0.3035310^{-1}

At the lapse of a time interval equal to unit $\boldsymbol{\theta}$ = $\sqrt{5}^{\pm 1}$, the core radius became a number inverse to invariable $\mathbf{W}_0\boldsymbol{v_0}$. Upon that:

1) The product of core radius $\boldsymbol{\varphi\omega}$ by invariable Wθ presents a value reciprocal to $\theta = \sqrt{\mathbf{5}}$:

$$\boldsymbol{\varphi\omega}\times\mathbf{W}\boldsymbol{\vartheta} = \sqrt{\mathbf{3\Phi\times\sqrt{5}}}^{\,-1}\times\sqrt{3\Phi\times\sqrt{5}^{-1}} = \sqrt{\mathbf{5}}^{-1}$$

0.3035310... x 1.4733704 = 0.4472136

2) The product of "interval θ" by the core-building invariable and by the core radius equals the Unit:

$$\mathbf{W}\boldsymbol{\vartheta}\times\boldsymbol{\varphi\omega}\times\sqrt{\mathbf{5}} = \mathbf{1}$$

3) Angle β is prerequisite for emergence of the Third invariable. Angle 2β = 52°37' x 2 = 105°14'. The symmetry-of-similarities space is built by angle 2α = 104°40'. The intramolecular bond angle of water molecule lies in the range 104° to 105° (Figs. 1.2; 7.5). Water is life. Thus unfolds the biological sense of the "Unit Φ" sphere.

4) The difference of squares of the second and third invariables is equal to 1.3708. In quantum physics, number $\frac{1}{\alpha}$ is *energy quantum***,** an invariable of thin structure ($\frac{1}{\alpha}$ = 1.3703).

The rhythm of expansion (a step: from sphere $\boldsymbol{\tau\omega}$ = $\mathbf{2}\sqrt{\left(\mathbf{3\Phi\times\sqrt{5}}\right)^{-1}}$ to sphere ***ab*** = $\sqrt{1}$; and from sphere ***ab*** = $\sqrt{1}$ to sphere AB = $\sqrt{\mathbf{5}}$) is comparable to the energy quantum.

Trebling and increasing number Φ by the factor **θ** = $\sqrt{\mathbf{5}}$ is an event: it involves modification of the space (i.e. space-time) structure. Mathematical modeling shows that spheres AB**, ab** and the third sphere $\boldsymbol{\tau\omega}$ are **structures of inverse integers existing on opposite sides of time interval θ.** Here is the essence of the Point of origin geometrical model: the innumerable multitude of spheres representing the symmetry-of-pairs law exists simultaneously, which is proved by the Second Pythagorean Theorem.

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[5] **Proof** (see Fig. 15, 2-3). From similarity between $\mathbf{W_0}$k and $\varphi\omega' v_{0_0}$ follows: $\varphi\omega'$ = $\sqrt{\left(\mathbf{3\Phi\times\sqrt{5}}\right)^{-1}}$ = 0.3035310..

$\mathbf{W}_0\boldsymbol{v_0}:\boldsymbol{\tau'\omega}$ = 1.4733704 : 0.3035310 = 3Φ.

1.4733704 × $\sqrt{5}$ = 3.2945564... = 0.3035310^{-1}.

П Р И Л О Ж Е Н И Я

A P P E N D I C E S

Таблица1. Уравнение симметрии пар.

Симметрия и антисимметрия чисел и знаков

Table 1. *Symmetry-of-pairs equation.*

Symmetry and antisymmetry of numbers and signs

Вид симметрии Symmetry type		$Ф^{+1}$				$Ф^{-1}$				Усл. обознач. Legend
a	Симметрия чисел Symmetry of numbers	● □	□ ●	□ ●	● □	● □	□ ●	□ ●	● □	● – число N, кратно 1 number N, Aliquot of 1
b	Антисимметрия чисел Antisymmetry of numbers	●	□	□	●	●	□	□	●	
		□	●	●	□	□	●	●	□	□ - число **θ**, кратно √5
c	Симметрия и анти–симметрия знаков Symmetry and anti-symmetry of signs	+		–		–		+		□ number **θ**, Aliquot of √5

Таблица2. Уравнение симметрии пар.

Поворотные симметрии

Table 2. Symmetry-of-pairs equation. Rotational symmetries

	Поворотные оси симметрии второго порядка Two-fold axes of rotational symmetry				Условные обозначения Legend
Единицы Units	**Ед. 1** ● Unit **1**	**Ед. 2** ● Unit **2**	**Ед.3** ● Unit **3**	**Ед.4** ● Unit **4**	● ось симметрии. ↺ ось анти симметрии
Звенья Links	↺		↺		
Структура из 2 звеньев 2-link structure	↺				● axis of symmetry ↺ axis of antisymmetry

Таблица3. Пятнадцать примеров решения уравнения симметрии пар (УСП) на сфере. Размеры для построения сферы в масштабе *1= 50 мм.* (См. рис. 2,3,5-8).

Table 3. Fifteen samples of plotting the symmetry-of-pairs equation (SPE) on a sphere Dimensions for building a sphere to scale: *1= 50 mm* (see Figs. 2,3,5-8)

№№ УСП SPE	Уравнение симметрии пар (УСП) Symmetry-of-pairs Equation (SPE)	Диаметр сферы 𝒜ℬ Sphere dia. 𝒜ℬ $\sqrt{A^2+B^2}$	Единица меры в мм Unit of measure mm	Размер на чертеже, в мм. Dimensions on drawing, mm				$\frac{\alpha\sqrt{5}+A}{B+\beta\sqrt{5}}$	
				A	$\alpha\sqrt{5}$	**B**	$\beta\sqrt{5}$		
1 B/A	2	3	4	5	6	7	8	9	10 β / α
A<B **1** 2,0	$\frac{\sqrt{5}+1}{2+0\sqrt{5}}=\frac{2-0\sqrt{5}}{\sqrt{5}-1}$	$\sqrt{5}$ 2,236	50	50	111,80	100	0	$\frac{161{,}8}{100.0}$	0
2 1,5	$\frac{8\sqrt{5}+10}{15+\sqrt{5}}=\frac{15-\sqrt{5}}{8\sqrt{5}-10}$	$\sqrt{325}$	6,202	62,02	110,94	93,03	13,87	$\frac{172.9}{106{,}9}$	0,125
3 1,375	$\frac{6\sqrt{5}+8}{11+\sqrt{5}}=\frac{11-\sqrt{5}}{6\sqrt{5}-8}$	$\sqrt{185}$	8,22	65,76	110,28	90,42	18,38	$\frac{176.0}{108{,}8}$	0,166
4 1,166	$\frac{4\sqrt{5}+6}{7+\sqrt{5}}=\frac{7-\sqrt{5}}{4\sqrt{5}-6}$	$\sqrt{85}$	12,12 7	72,76	108,47	84,89	27,12	$\frac{181.23}{112{,}0}$	0,250
5 1,048	$\frac{13\sqrt{5}+21}{22+4\sqrt{5}}\ \frac{22-4\sqrt{5}}{13\sqrt{5}-21}$	$\sqrt{925}$	3,676	77,2	106,86	80,87	32,88	$\frac{184.1}{113.7}$	0,307
A>B **6** 0,846	$\frac{7\sqrt{5}+13}{11+3\sqrt{5}}=\frac{11-3\sqrt{5}}{7\sqrt{5}-13}$	$\sqrt{290}$	6,565	86,12	103,7	72,87	44,44	$\frac{188{,}1}{110{,}2}$	0,428
7 0,75	$\frac{2\sqrt{5}+4}{3+\sqrt{5}}=\frac{3-\sqrt{5}}{2\sqrt{5}-4}$	$\sqrt{25}$	22,36 1	89,44	100	67,0	50	$\frac{189{,}4}{117{,}0}$	0,500
8 0,636	$\frac{5\sqrt{5}+11}{7+3\sqrt{5}}=\frac{7-3\sqrt{5}}{5\sqrt{5}-11}$	$\sqrt{170}$	8,574	94,3	95,86	60	57,5	$\frac{190{,}2}{117{,}5}$	0,600
9 0,529	$\frac{7\sqrt{5}+17}{9+5\sqrt{5}}=\frac{9-5\sqrt{5}}{7\sqrt{5}-17}$	$\sqrt{370}$	5,812 4	98,81	90,98	52,3	65	$\frac{189{,}8}{117{,}3}$	0,714
10 0,437	$\frac{6\sqrt{5}+16}{7+5\sqrt{5}}=\frac{7-5\sqrt{5}}{6\sqrt{5}-16}$	$\sqrt{305}$	6,402	102,4	85,89	44,81	71,57	$\frac{188{,}3}{116{,}4}$	0,833
11 0,333	$\frac{\sqrt{5}+3}{1+\sqrt{5}}=\frac{1-\sqrt{5}}{\sqrt{5}-3}$	$\sqrt{10}$	35,35	106,1	79,045	35,35	79,04	$\frac{185{,}1}{114{,}3}$	1,000,
12 0,214	$\frac{4\sqrt{5}+14}{3+5\sqrt{5}}=\frac{3-5\sqrt{5}}{4\sqrt{5}-14}$	$\sqrt{205}$	7,808	109,3	69,83	23,42	87,30	$\frac{179{,}1}{110{,}7}$	1,250
13 0,125	$\frac{2\sqrt{5}+8}{1+3\sqrt{5}}=\frac{1-3\sqrt{5}}{2\sqrt{5}-8}$	$\sqrt{65}$	13,86 7	110,9	62,015	13,87	93,02	$\frac{172{,}9}{106{,}9}$	1,500
14 0,077	$\frac{3\sqrt{5}+13}{1+5\sqrt{5}}=\frac{1-5\sqrt{5}}{3\sqrt{5}-13}$	$\sqrt{170}$	8,575	111,5	57,52	8,575	95,87	$\frac{169}{104{,}4}$	1,666
15 0,043	$\frac{5\sqrt{5}+23}{1+9\sqrt{5}}=\frac{1-9\sqrt{5}}{5\sqrt{5}-23}$	$\sqrt{530}$	4,856	111,8	54,3	4,85	97,73	$\frac{166}{102{,}6}$	1,800

Таблица 4. Симметрииi $(\frac{+}{-} = \frac{+}{-})$ и закон обратных дихотомий:

Table 4. Symmetry and the law of contrary dichotomies.

$A = 1/2\,(5\beta + B)$ $B = 1/2\,(5\propto - A)$
$\propto = 1/2\,(\beta + B)$ $\beta = 1/2\,(\propto - A)$

ЛЕВАЯ ЧАСТЬ ↔ ПРАВАЯ ЧАСТЬ
Left part Right part

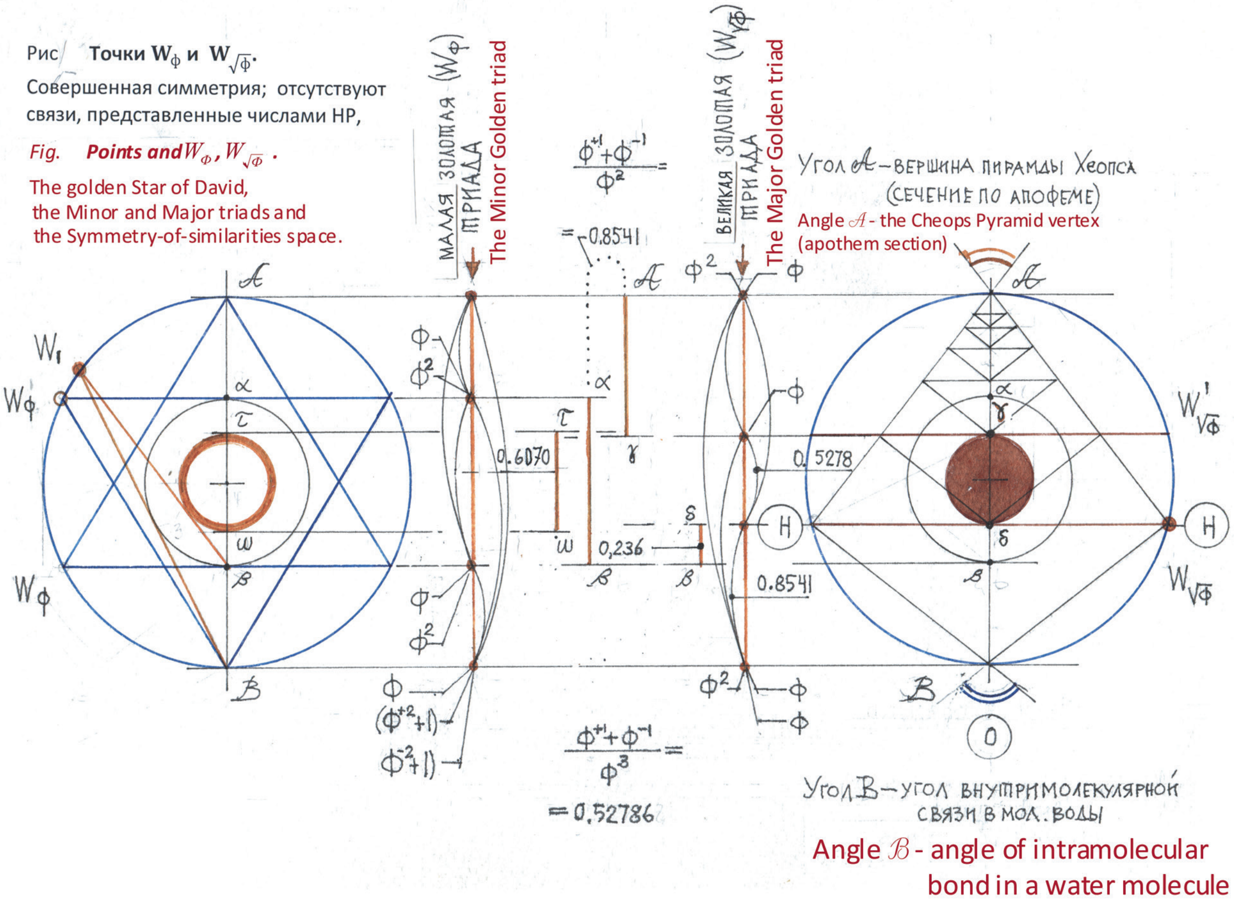
УСП -16 SPE-16 $\frac{17\sqrt{5}+15}{35-\sqrt{5}} = \frac{35+\sqrt{5}}{17\sqrt{5}-15}$	$\frac{53{,}013}{32{,}763} = \frac{37{,}236}{23{,}013} = \phi$ 1.618 = 1,618 1.618	15=0,5(-5+35)=15 A 17=0,5(35-1) = 17 $\propto$ 35=0.5(85-15)=35 B -1 =0.5(15-17)=-1 β	-15=0,5(5-35)= -15 A +1= 0,5(17-15)=+1 $\propto$ -15=0,5(5-35) =-15 B **+17**=0,5(+1-35)=**-17** β
УСП - 17 SPE-17 $\frac{7\sqrt{5}+5}{15-\sqrt{5}} = \frac{15+\sqrt{5}}{7\sqrt{5}-5}$	$\frac{20{,}652}{12{,}764} = \frac{17{,}236}{10{,}652} = \phi$ 1.618 = 1,618 1.618	5=0,5(-5+15)=5 A 7=0,5(-1+15) = 7 $\propto$ 15=0.5(35-5)=15 B **-1** =0.5(7-5)= **+1** β	15=0,5(35-5)=15 A 1=0,5(-5+7) = 1 $\propto$ -5=0.5(5-15)=-5 B +7=0.5(15-1)=+7 β
УСП - 18 SPE-18 $\frac{13\sqrt{5}+7}{29-3\sqrt{5}} = \frac{29+3\sqrt{5}}{13\sqrt{5}-7}$	$\frac{36{,}068}{22{,}292} = \frac{35{,}708}{22{,}069} = \phi$ 1.618 = 1,618 1.618	7=0,5(-15+29) = 7 A 13=0,5(-3+29) =13 $\propto$ 29=0.5(65-7)=29 B **-3=** 0.5(13-7)= **+3** β	29=0,5(65-7) = 29 A 3=0,5(-7+13) = 3 $\propto$ -7=0.5(15-29)=-7 B **+13** =0.5(3-29)=**-13** β
УСП - 19 SPE-19 $\frac{67\sqrt{5}+53}{141-7\sqrt{5}} = \frac{141+7\sqrt{5}}{67\sqrt{5}-53}$	$\frac{202{,}816}{125{,}347} = \frac{156{,}652}{96{,}816} = \phi$ 1.618 = 1,618 1.618	53=0,5(-35+141) = 53 A 67= 0,5(-7+141) = 67 $\propto$ 141= 0.5 (335-53)= 141 B 7 =0.5(67-53)= 7 β	141=0,5(335-53) =141 A 7=0,5(67-53) = 7 $\propto$ -53= 0,5(35-141)=-53 B **+67**=0.5(+7-141)= **-67** β
УСП - 9 SPE-9 $\frac{7\sqrt{5}+17}{9+5\sqrt{5}} = \frac{9-5\sqrt{5}}{7\sqrt{5}-17}$	УСП - 8 $\frac{5\sqrt{5}+11}{7+3\sqrt{5}} = \frac{7-3\sqrt{5}}{5\sqrt{5}-11}$	УСП - 16 $\frac{3\sqrt{5}+5}{10+4\sqrt{5}} = \frac{10-4\sqrt{5}}{3\sqrt{5}-5}$	УСП - 2 $\frac{8\sqrt{5}+10}{15+\sqrt{5}} = \frac{15-\sqrt{5}}{8\sqrt{5}-10}$
УСП - 1 SPE-1 $\frac{\sqrt{5}+1}{2+0\sqrt{5}} = \frac{2-0\sqrt{5}}{\sqrt{5}-1}$			

ИЛЛЮСТРАЦИИ К

I ЧАСТИ

ILLUSTRATIONS

PART I



Золотые триады.
Golden triads.

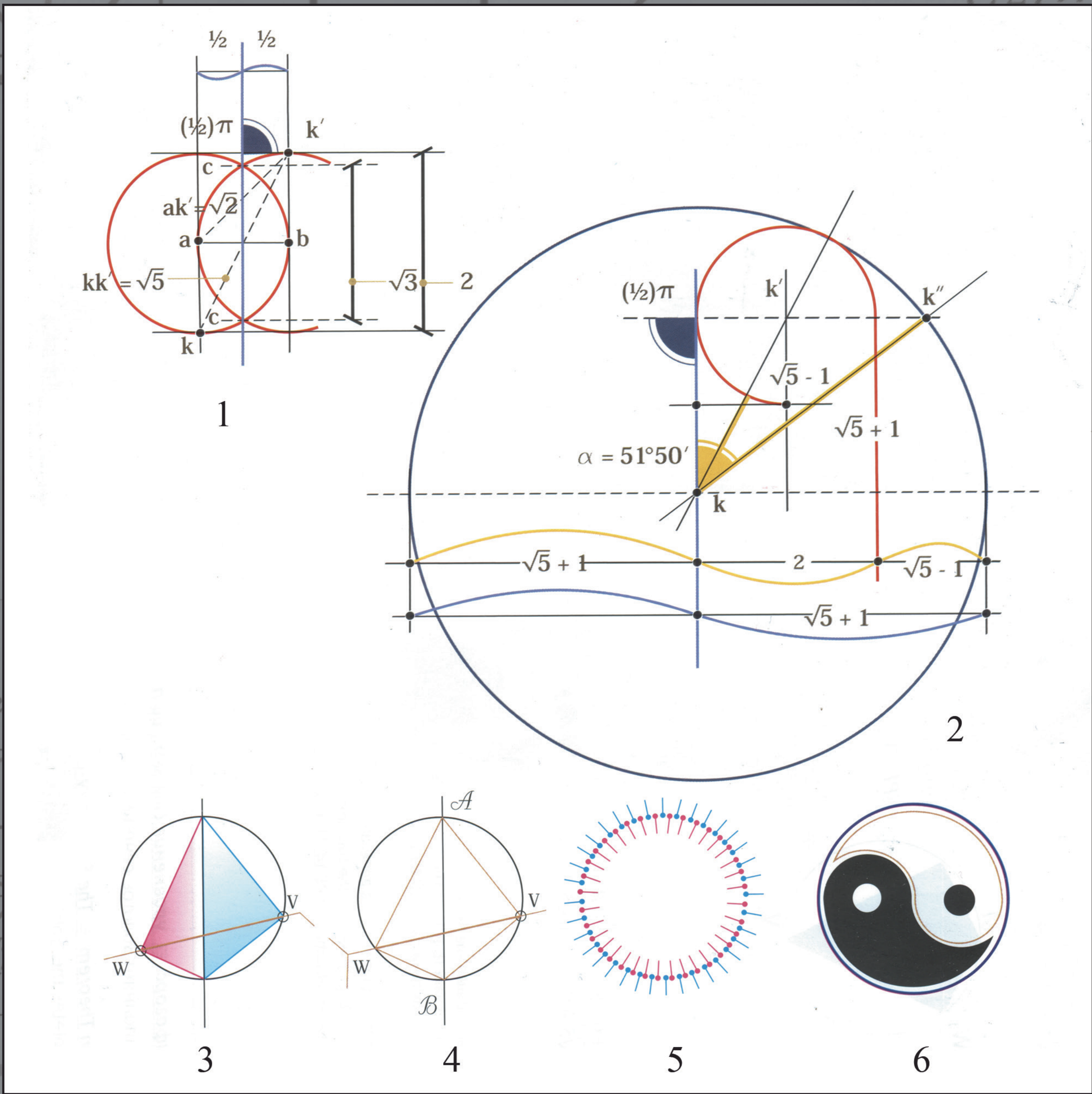


Рисунок
Figure 1

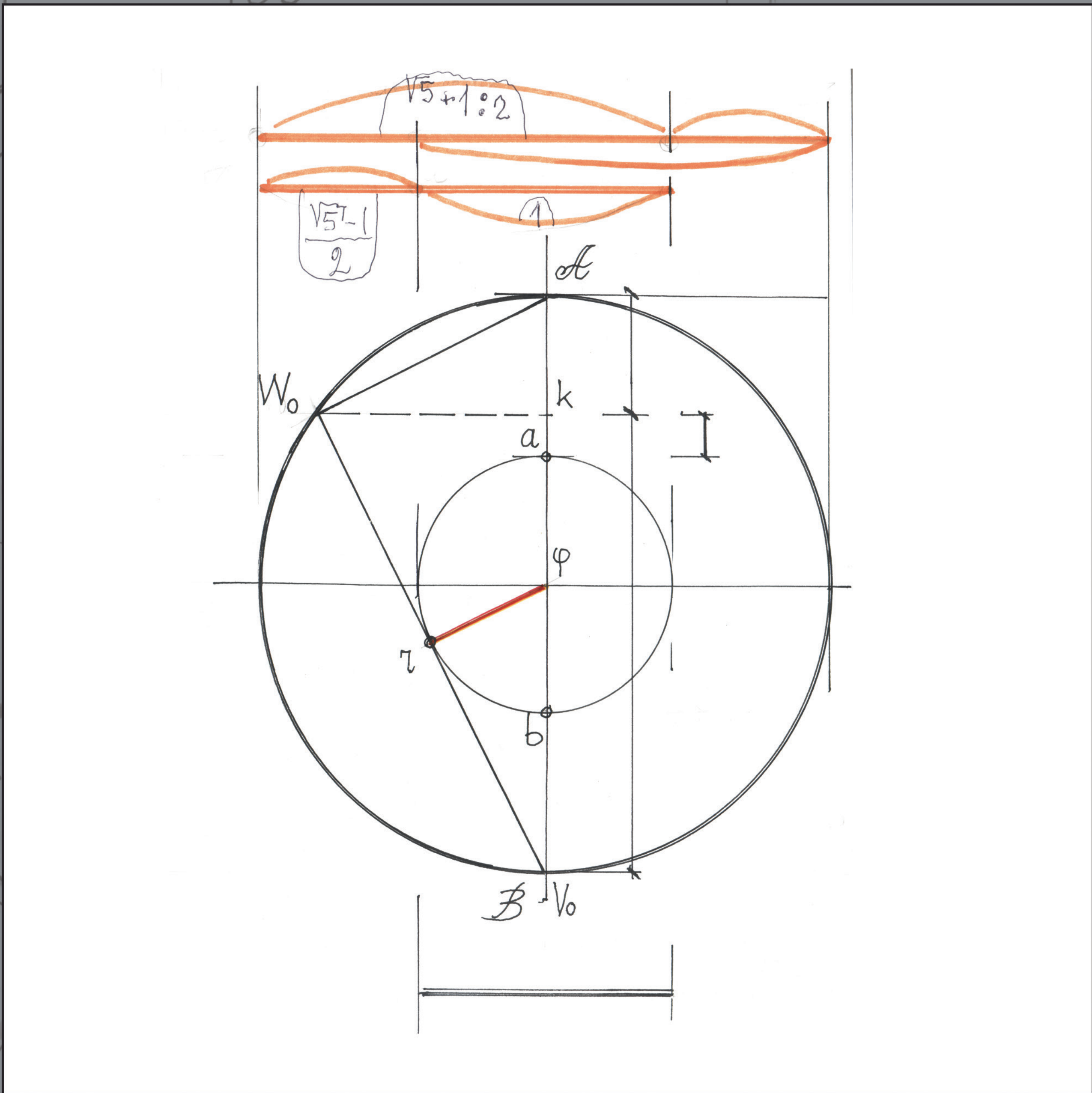
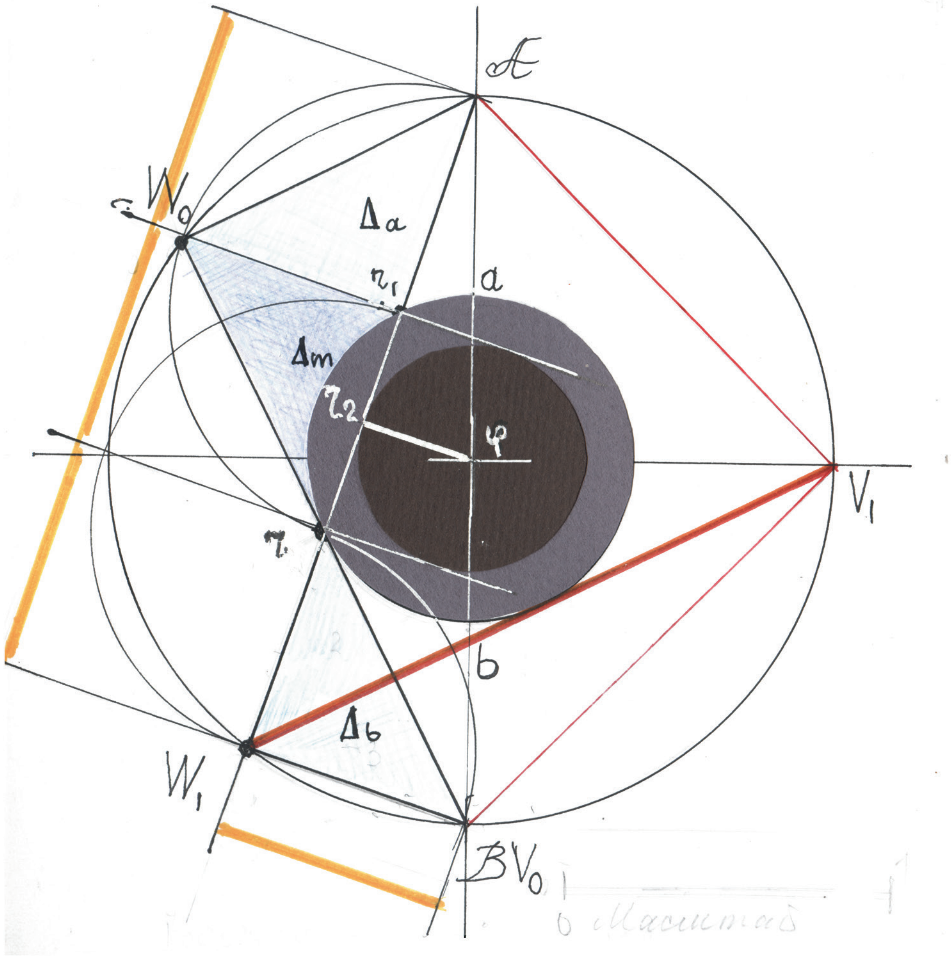


Рисунок
Figure 2.1



$$\frac{\sqrt{5}+1}{2+0\sqrt{5}}=\frac{2-0\sqrt{5}}{\sqrt{5}-1} \quad = \quad \frac{\sqrt{5}+3}{1+\sqrt{5}}=\frac{1-\sqrt{5}}{\sqrt{5}-3}$$

Рисунок
Figure 2.2

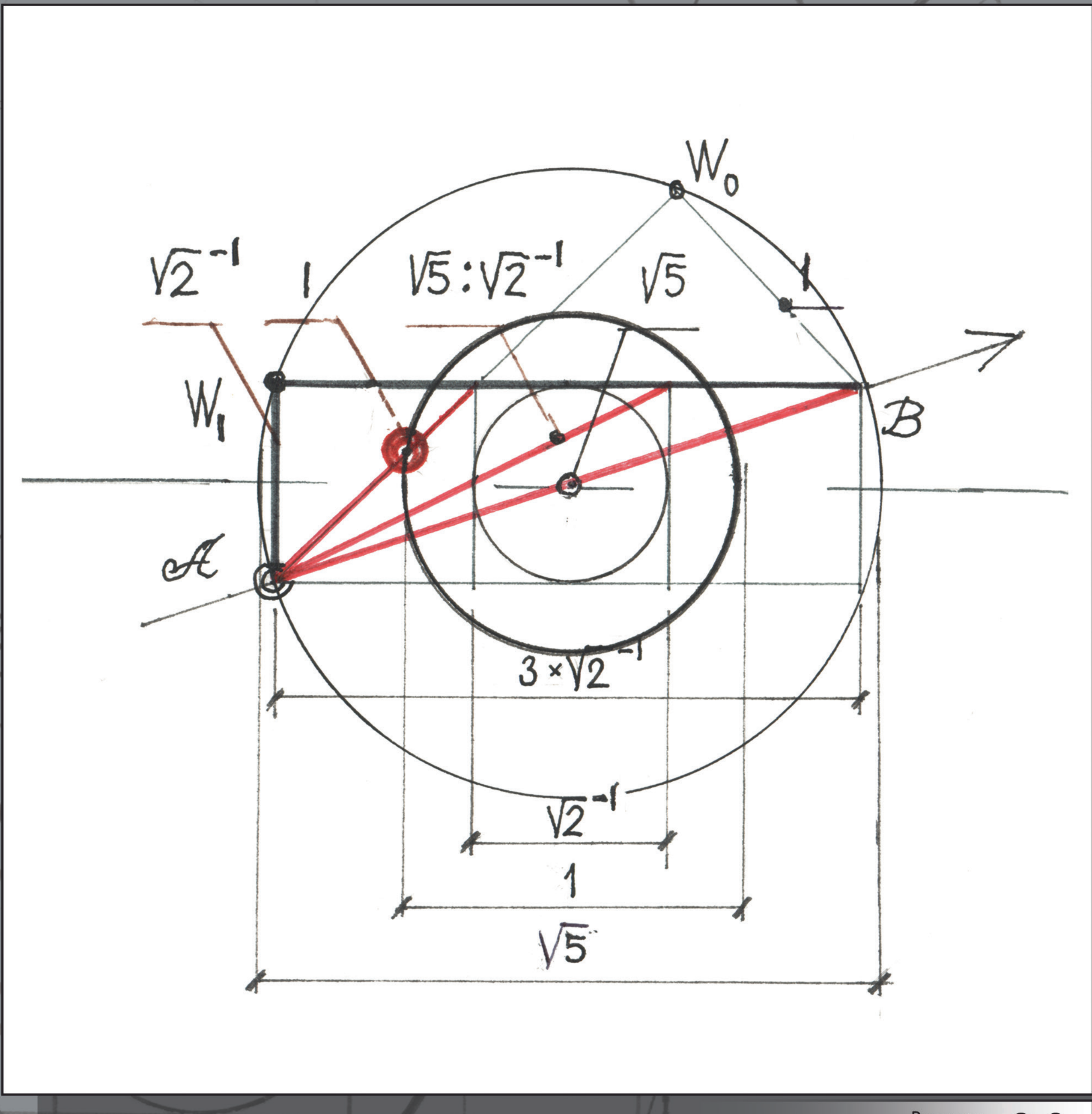


Рисунок
Figure
2.3

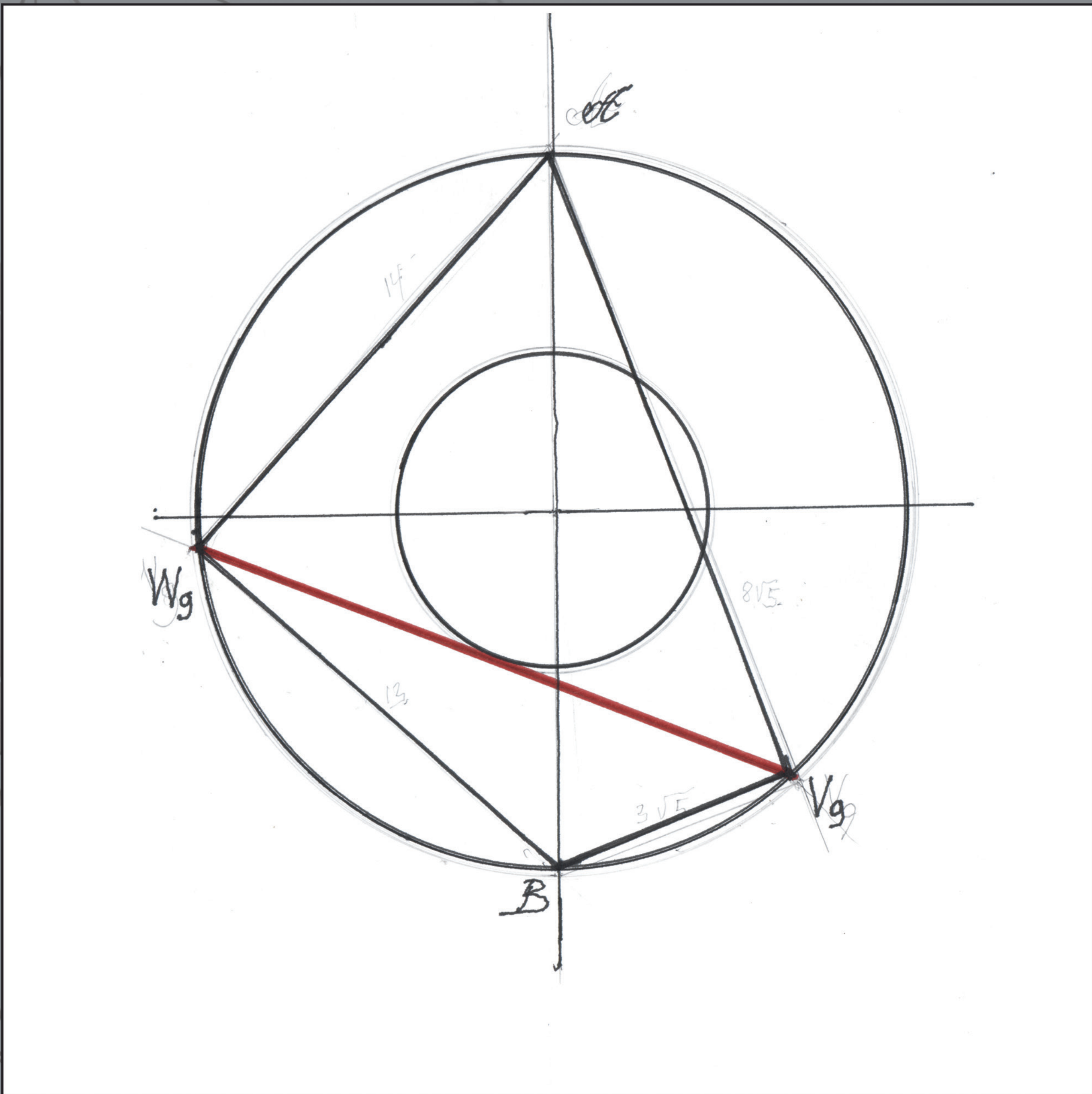


Рисунок
Figure 3.1

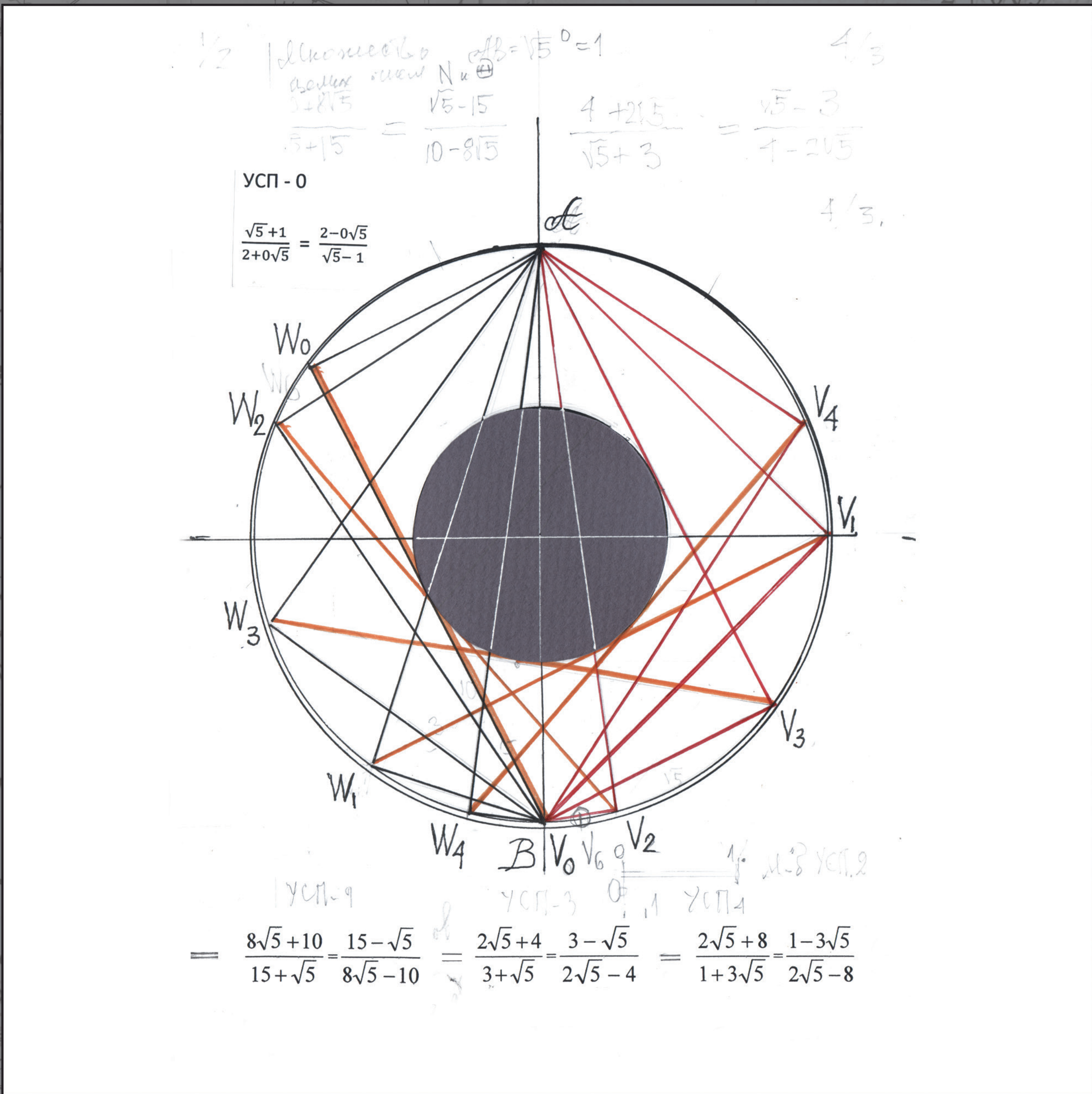
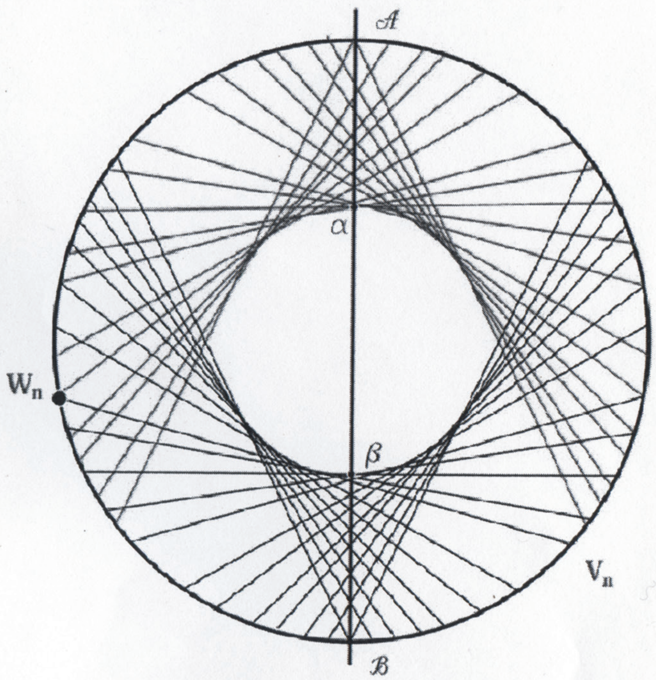


Рисунок
Figure 3.2



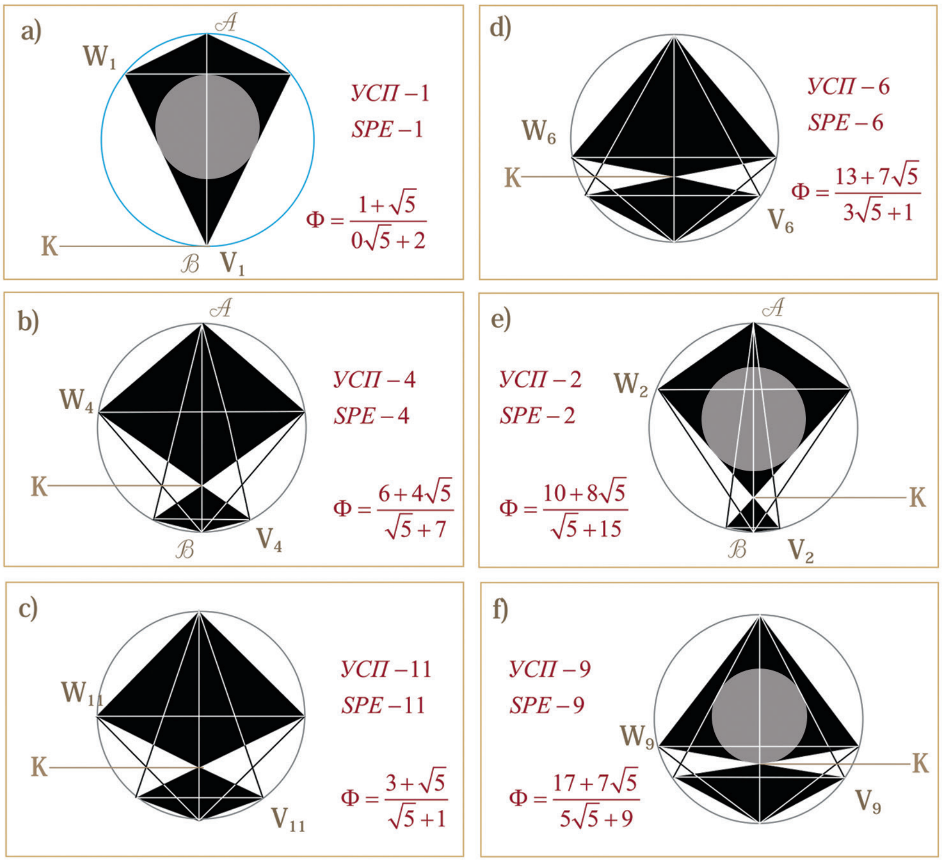
a)

Вторая константа
Second invariable

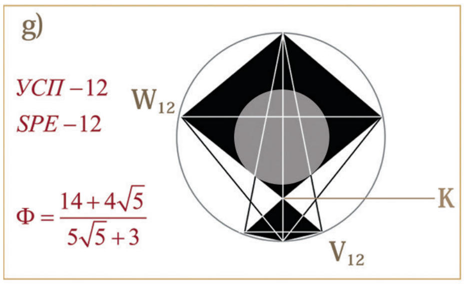
$$WV = \Phi^{+1} + \Phi^{-2} = \Phi^{+2} - \Phi^{-1} = 2$$

$\mathcal{AB} = \sqrt{5}$ $\alpha\beta = 1$

Рисунок
Figure 3.3



$$\Phi_n = \frac{A+\alpha\sqrt{5}}{b\sqrt{5}+B} = \frac{b\sqrt{5}-B}{A-\alpha\sqrt{5}} = \frac{-b\sqrt{5}+B}{-A+\alpha\sqrt{5}}$$



В случае «a» уравнение вырождено, β=0.
В остальных случаях (b-g) - это четырехбуквенные уравнения симметрии пар (УСП № 14,11 6,2,9,12).

In case «a» the equation is in a generate form, β=0.
In other cases (“b-g ”) we have the four-letter symmetry-of-pairs equations (SPE 14, 11, 6, 2, 9, 12).

Рисунок
Figure 4.1

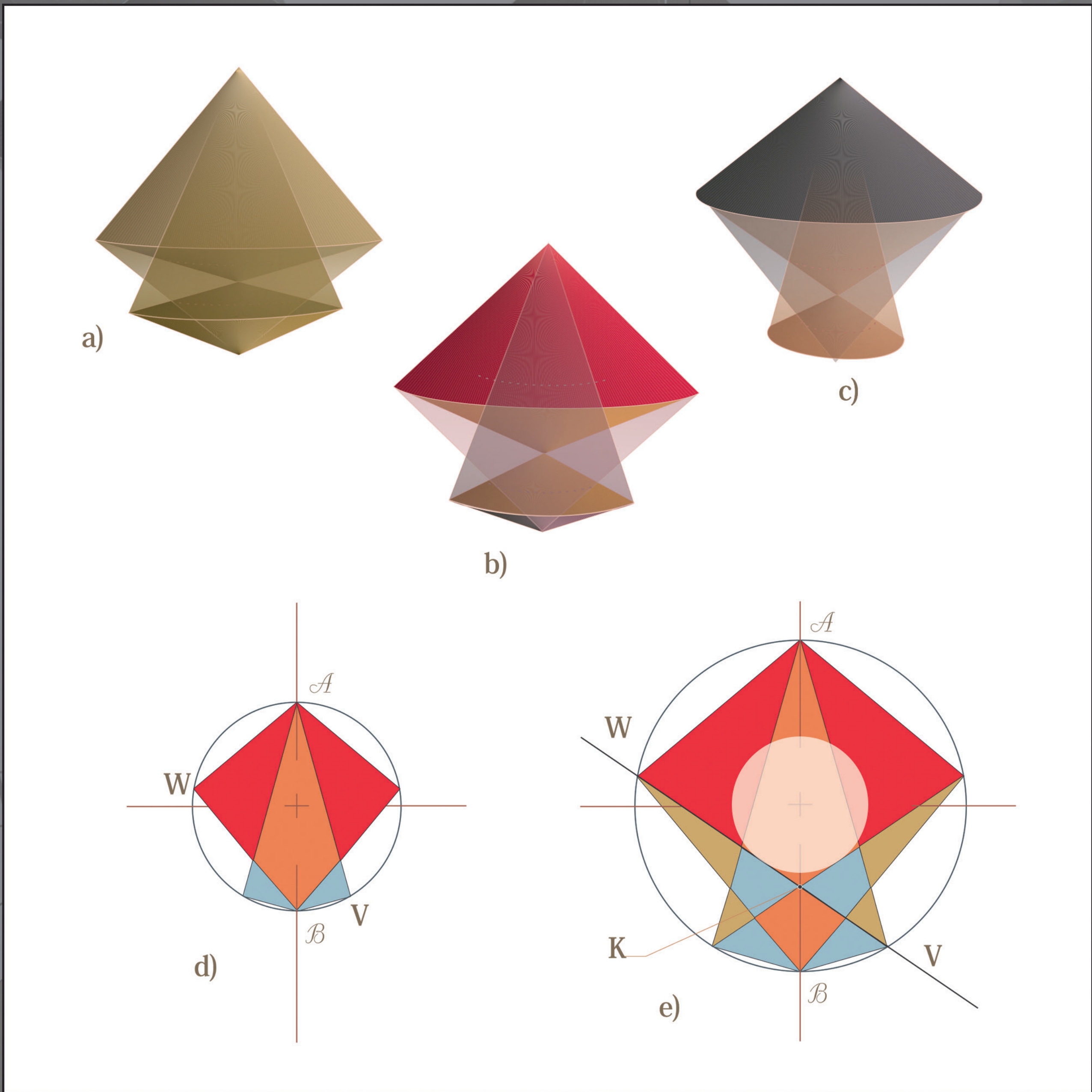


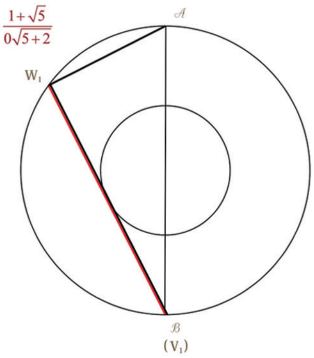
Рисунок
Figure 4.2

Рис. 3. Три пары конических тел вписаны в одно сферическое пространство.

Паре чисел НР 2:1 отвечают две пары чисел ряда θ – вырожденная $0\sqrt{5}:\sqrt{5}$ и пара $10:5$
a/ (вырожденное) b/ его двойник.

The pair of natural integers 2:1 correlates with two θ -series pairs: degenerate pair $0\sqrt{5}:\sqrt{5}$ and pair $10:5$
a/ (degenerate form) b/ its duplicate

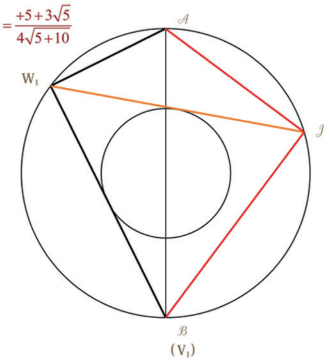
a) $\Phi = \dfrac{1+\sqrt{5}}{0\sqrt{5}+2}$



$\mathcal{AB} = \sqrt{5}$ $W_1V_1 = 2$ $W_1V_1 \equiv V_1\mathcal{B}$
Откуда следует $BV_1 = \beta = 0$
Whence it follows: $BV_1 = \beta = 0$

5.1

b) $\Phi = \dfrac{+5+3\sqrt{5}}{4\sqrt{5}+10}$

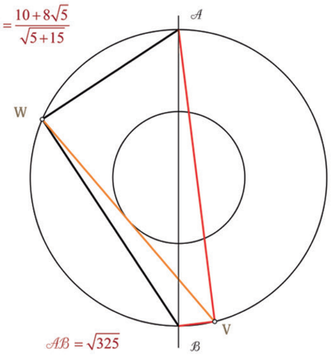


$\mathcal{AB} = 5\sqrt{5}$ $\mathcal{A}W_1 = 5$ $\mathcal{A}J = 3\sqrt{5}$ $\mathcal{B}J = 4\sqrt{5}$
Откуда следует $\mathcal{A}J : \mathcal{B}J = 3:4;$ $W_1B : \mathcal{A}W_1 = 2$
Whence it follows: $\mathcal{A}J : \mathcal{B}J = 3:4;$ $W_1B : \mathcal{A}W_1 = 2$

5.2

Паре чисел НР 3:2 отвечают две пары чисел ряда θ , пары $8\sqrt{5}:\sqrt{5}$ и пара $4\sqrt{5}:7\sqrt{5}$
A pair of natural integers 3:2 correlates with θ -series pairs: pair $8\sqrt{5}:\sqrt{5}$ and pair $4\sqrt{5}:7\sqrt{5}$

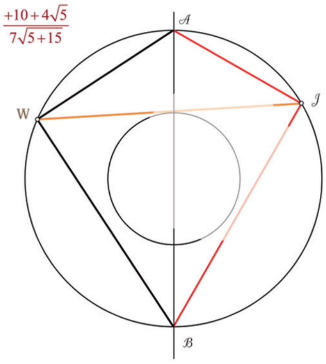
a) $\Phi = \dfrac{10+8\sqrt{5}}{\sqrt{5}+15}$



$\mathcal{AB} = \sqrt{325}$
$WV = 2/\sqrt{5}$
$\mathcal{B}V : V\mathcal{A} = 1:8;$
$\mathcal{A}W : W\mathcal{B} = 2:3$

5.3

b) $\Phi = \dfrac{+10+4\sqrt{5}}{7\sqrt{5}+15}$



$\mathcal{A}J : \mathcal{B}J = 4:7;$
$W\mathcal{B} : \mathcal{A}W = 3:2$

5.4

Рисунок
Figure 5.1,5.2,5.3,5.4

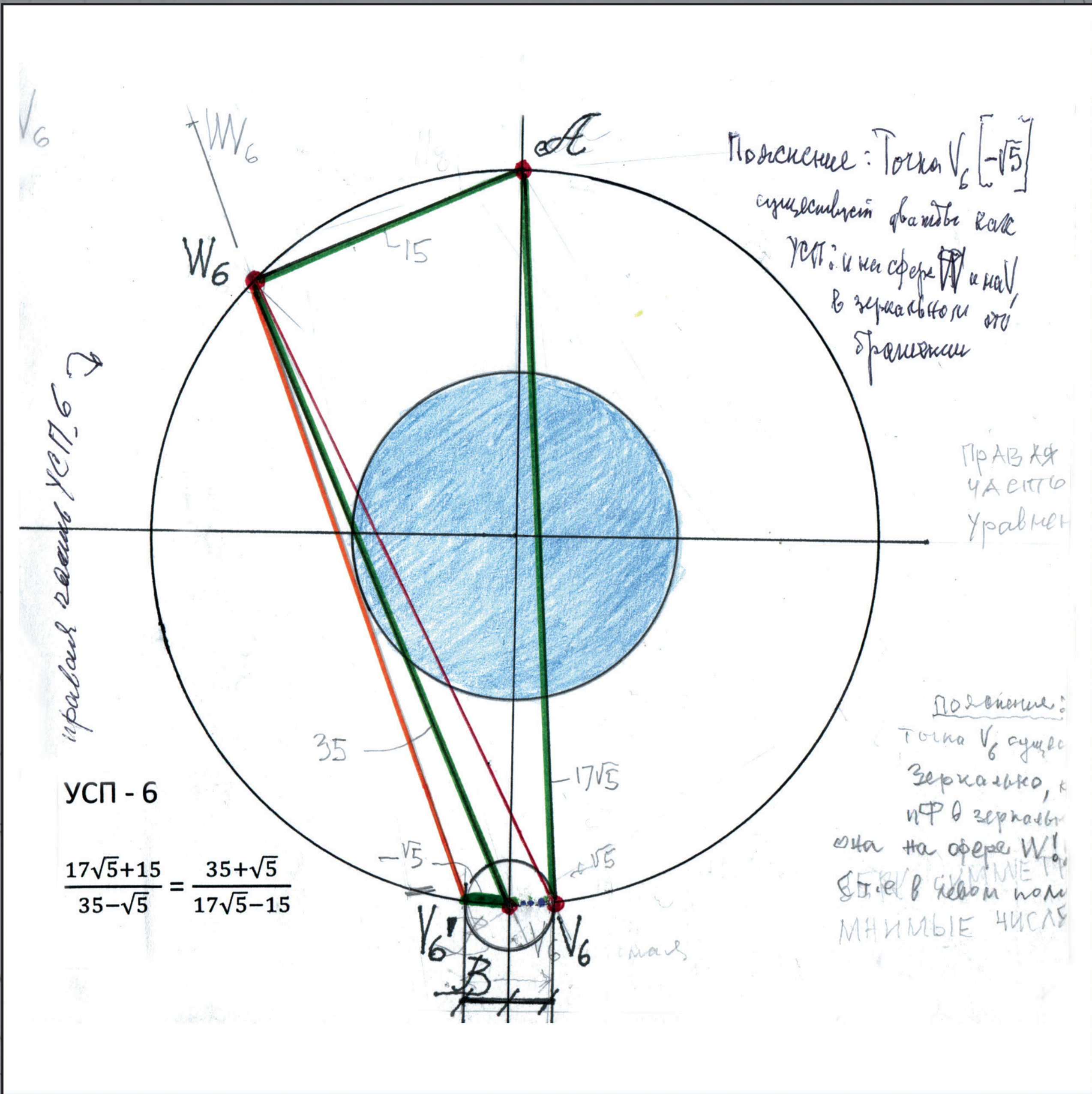


Рисунок
Figure
6.1

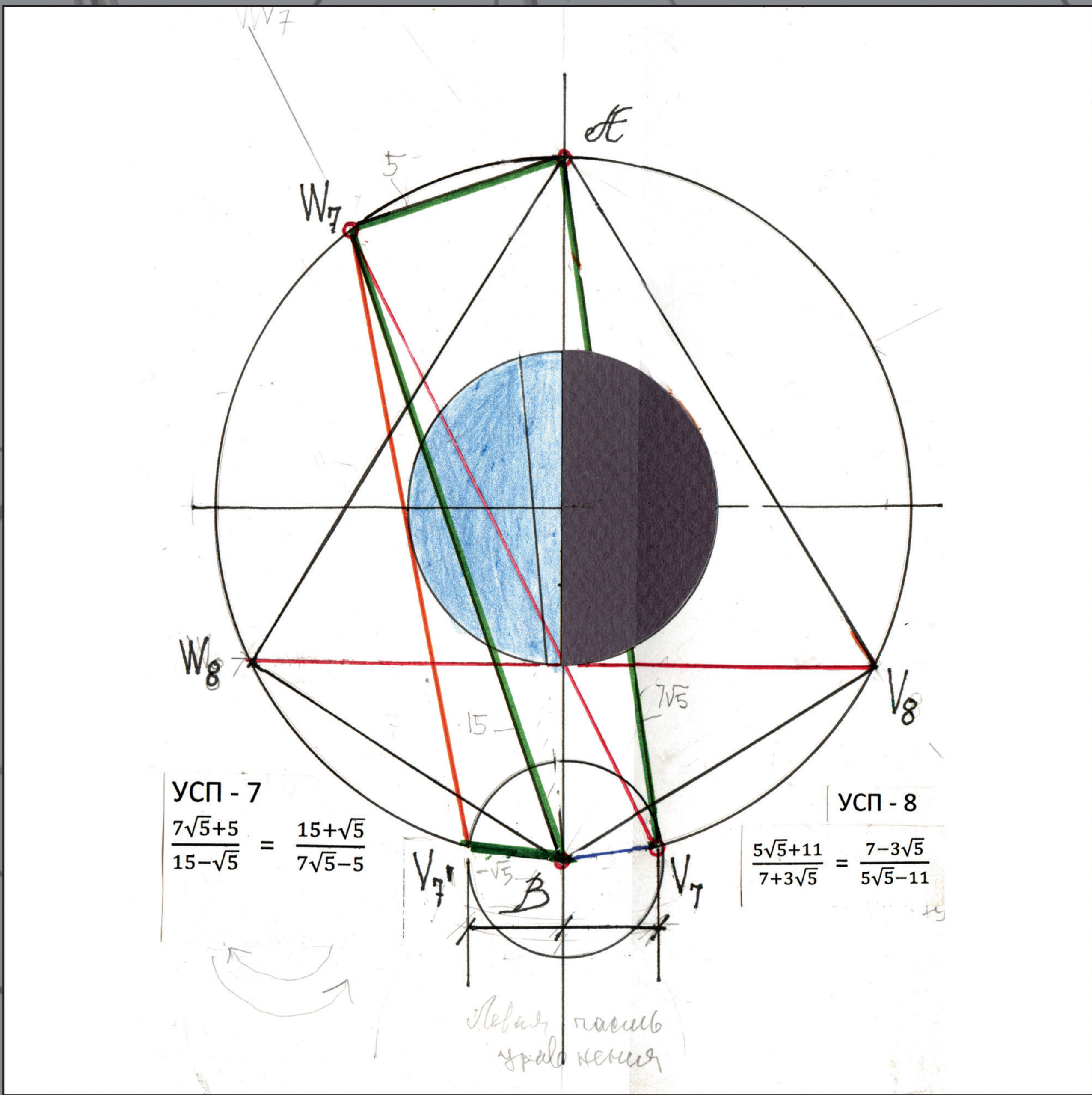


Рисунок
Figure
6.2

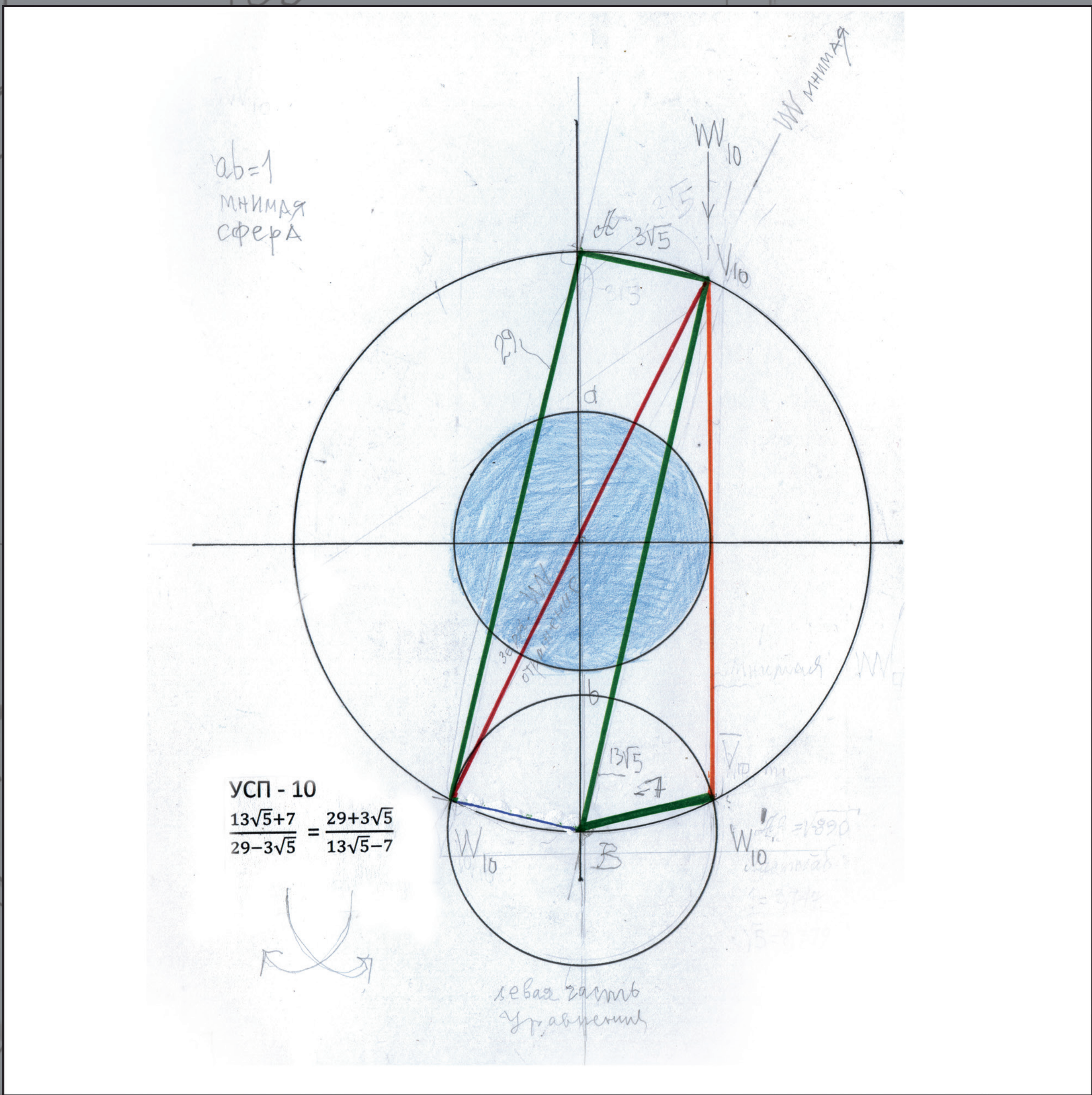


Рисунок
Figure
6.3

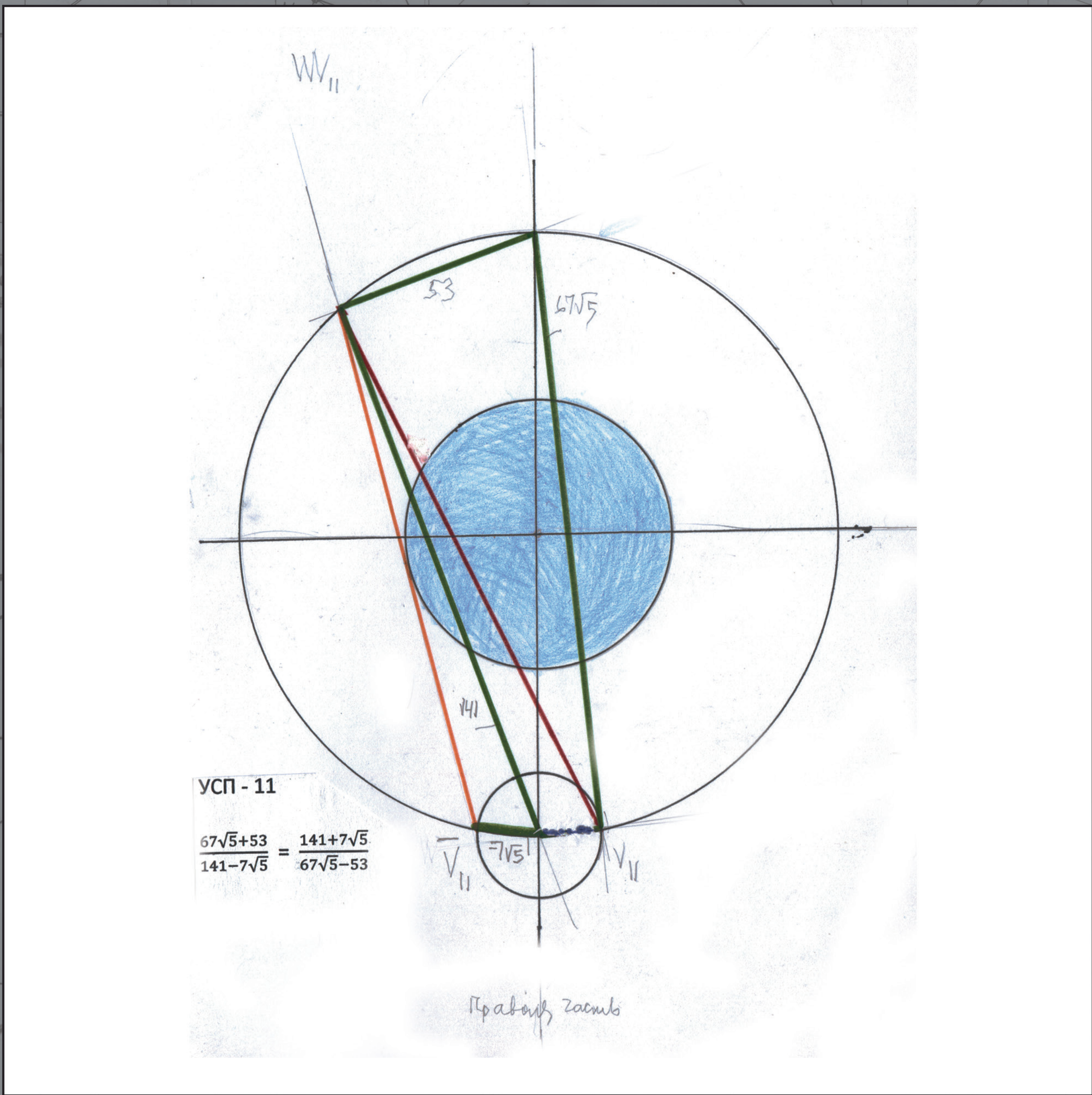


Рисунок
Figure
6.4

$$\Phi = \frac{\sqrt{5}+3}{1+\sqrt{5}}$$

УСП №11. Числа **НР** и числа ряда **θ** расположены на разных сферах. Сферы «меняются местами».
a/ Числа N на сфере $\mathcal{AB} = \sqrt{5}$ b/ Числа N на сфере $\sqrt{1}$.

SPE 11. Natural integers and θ-series numbers are arranged on different spheres. The spheres “change places”.
a/ Numbers N on sphere $\mathcal{AB} = \sqrt{5}$ b/ Numbers N on sphere $\sqrt{1}$

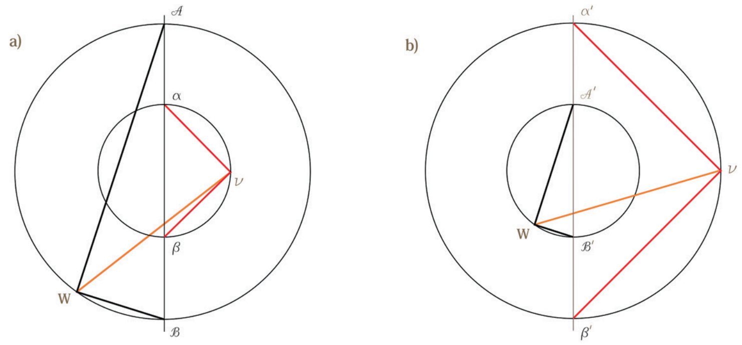


Рис. 13. ***Третья константа***
Fig. 13. ***The third invariable***

$$WJ = \sqrt{\frac{3\Phi}{\sqrt{5}}}\,\alpha\beta = \sqrt{\frac{3\Phi}{5\sqrt{5}}}\,\mathcal{AB}$$ ***или / or*** $$wV = \sqrt{\frac{3\Phi}{\sqrt{5}}}\,\mathcal{A}'\mathcal{B}' = \sqrt{\frac{3\Phi}{5\sqrt{5}}}\,\alpha'\beta'$$

7.1 7.2

УСП №2. Числа **НР** и числа ряда θ расположены на разных сферах. Сферы «меняются местами».
a/ Числа N на сфере $\mathcal{AB} = \sqrt{5}$. b/ Числа N на сфере $\mathcal{A}'\mathcal{B}' = \sqrt{1}$

$$\Phi = \frac{10+4\sqrt{5}}{7\sqrt{5}+15}$$

SPE 2. Natural integers and θ -series numbers are arranged on different spheres. The spheres “change places”.
a/ Numbers N on sphere $\mathcal{AB} = \sqrt{5}$. b/ Numbers N on sphere $\mathcal{A}'\mathcal{B}' = \sqrt{1}$

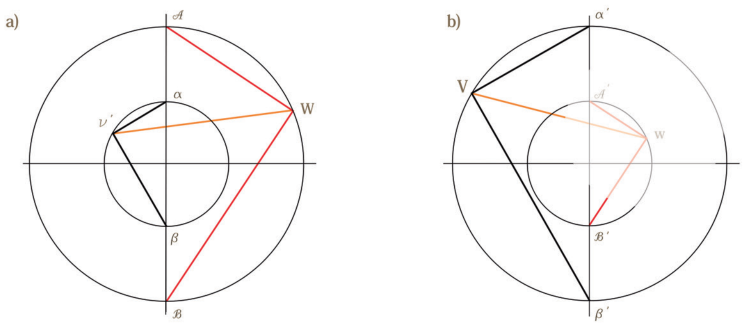


Рис. 14. ***Третья константа***
Fig. 14. ***The third invariable***

$$W\vartheta = \sqrt{\frac{3\Phi}{\sqrt{5}}}\,\alpha\beta = \sqrt{\frac{3\Phi}{5\sqrt{5}}}\,\mathcal{AB}$$ ***или / or*** $$wV = \sqrt{\frac{3\Phi}{\sqrt{5}}}\,\mathcal{A}'\mathcal{B}' = \sqrt{\frac{3\Phi}{5\sqrt{5}}}\,\alpha'\beta'$$

7.3 7.4

Рисунок
Figure 7.1,7.2,7.3,7.4

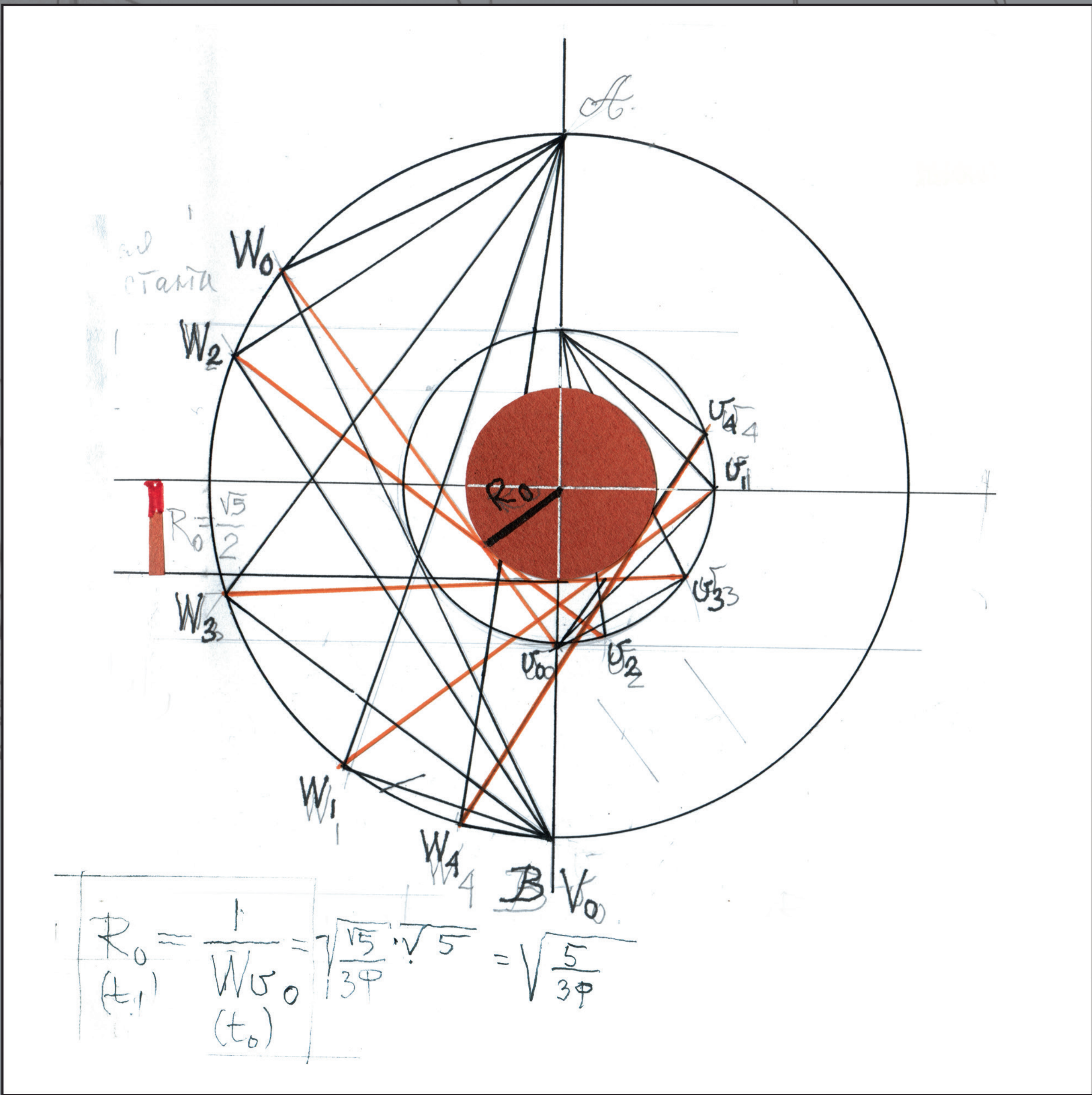
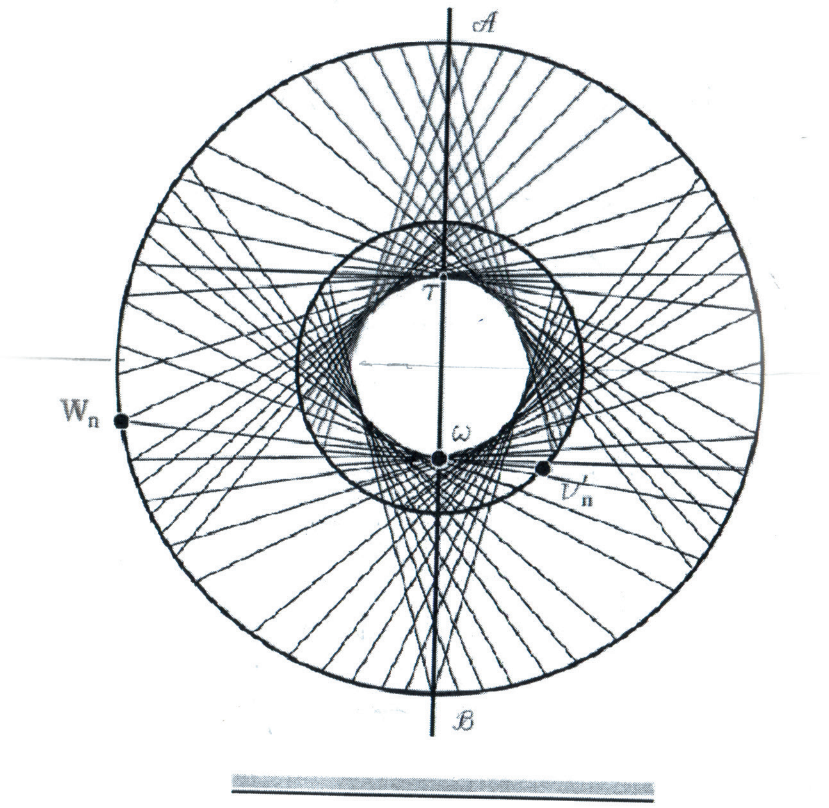


Рисунок
Figure 8.1

Третья константа
Third invariable

$$WJ = \left[\frac{\Phi^{3}+\Phi^{-1}}{\Phi^{+1}+\Phi^{-1}}\right]^{1/2} = \sqrt{\frac{3\Phi}{\sqrt{5}}}$$



b)

Третья константа
Third invariable

$$WV' = \left[\frac{\Phi^3+\Phi^{-1}}{\Phi^{+1}+\Phi^{-1}}\right]^{1/2} = \sqrt{\frac{3\Phi}{\sqrt{5}}}$$

Рисунок
Figure
8.2

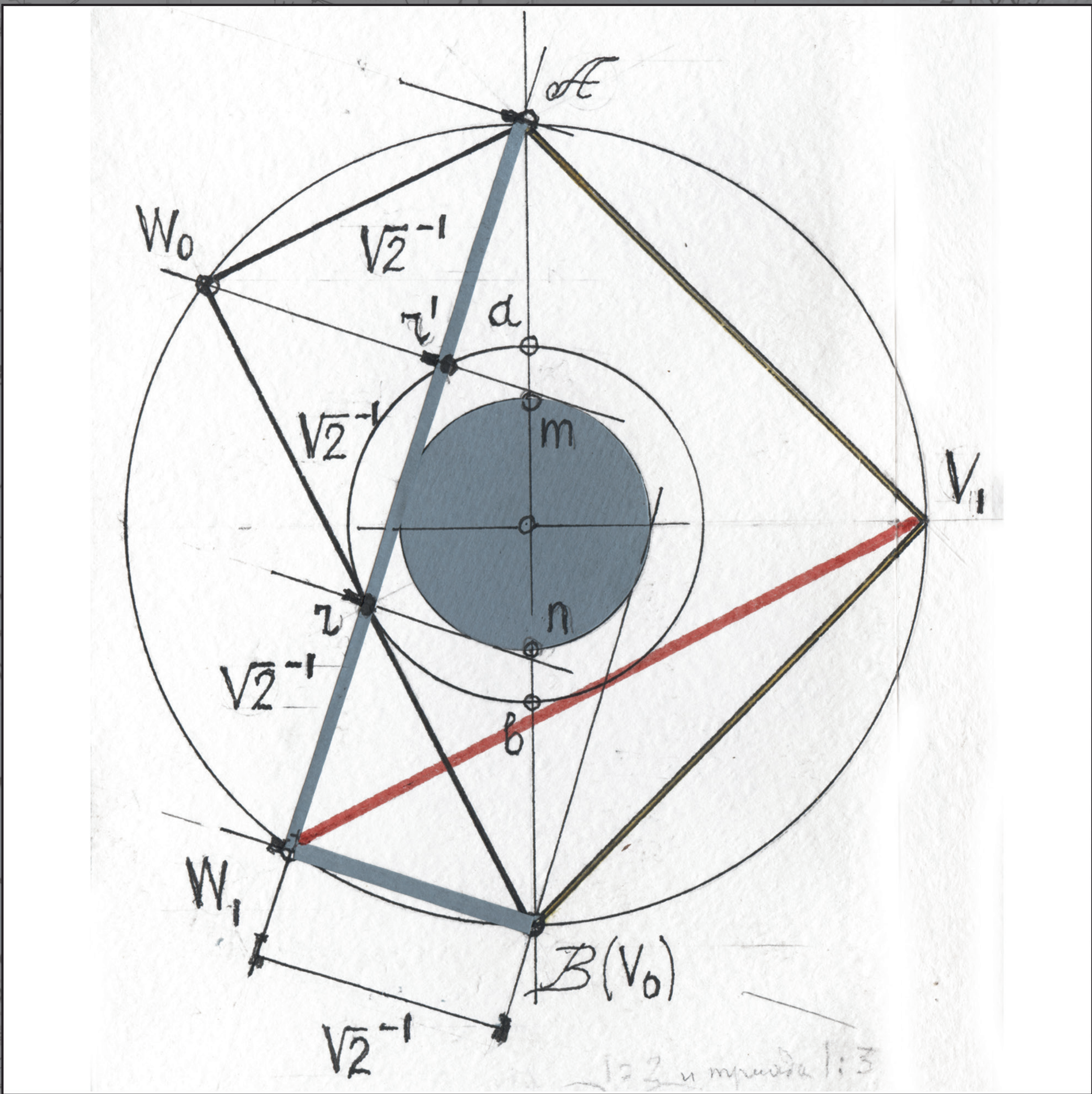


Рисунок
Figure 9.1

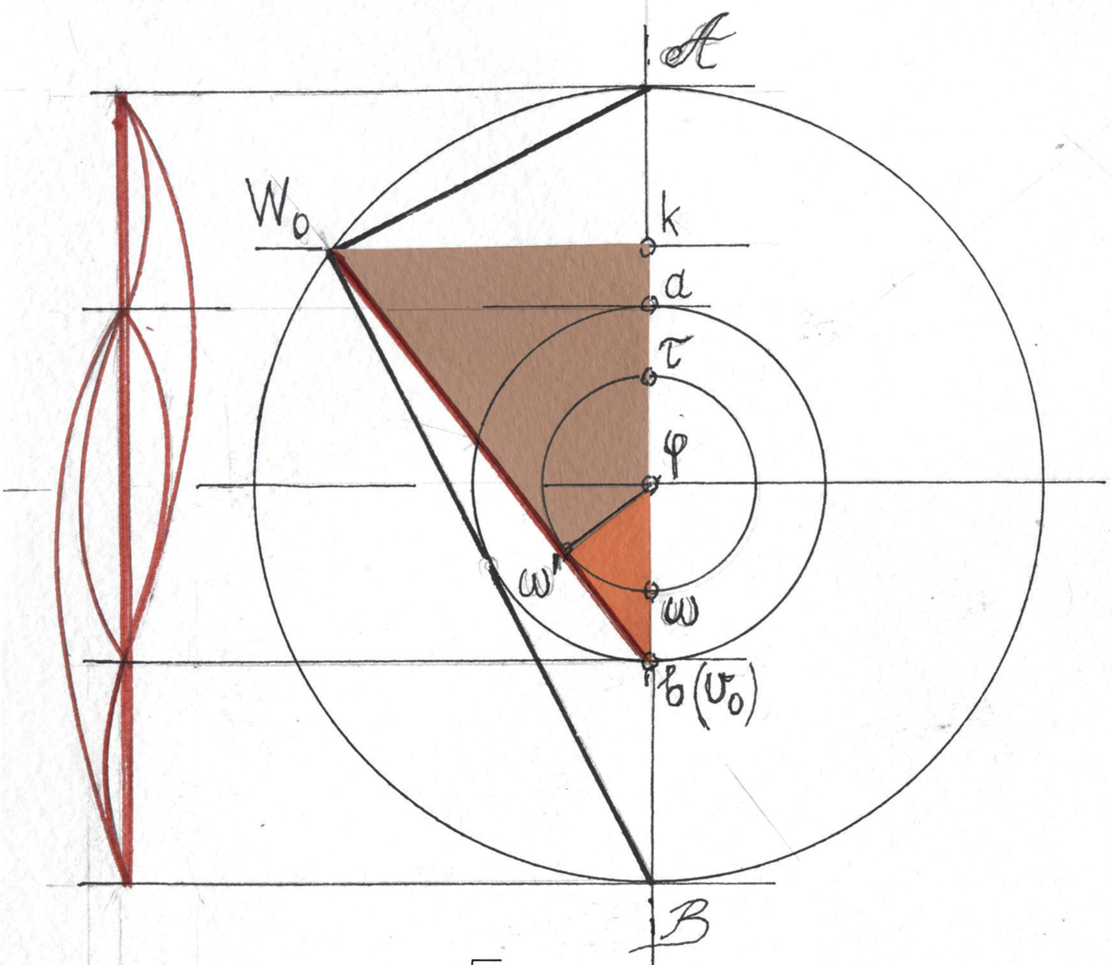
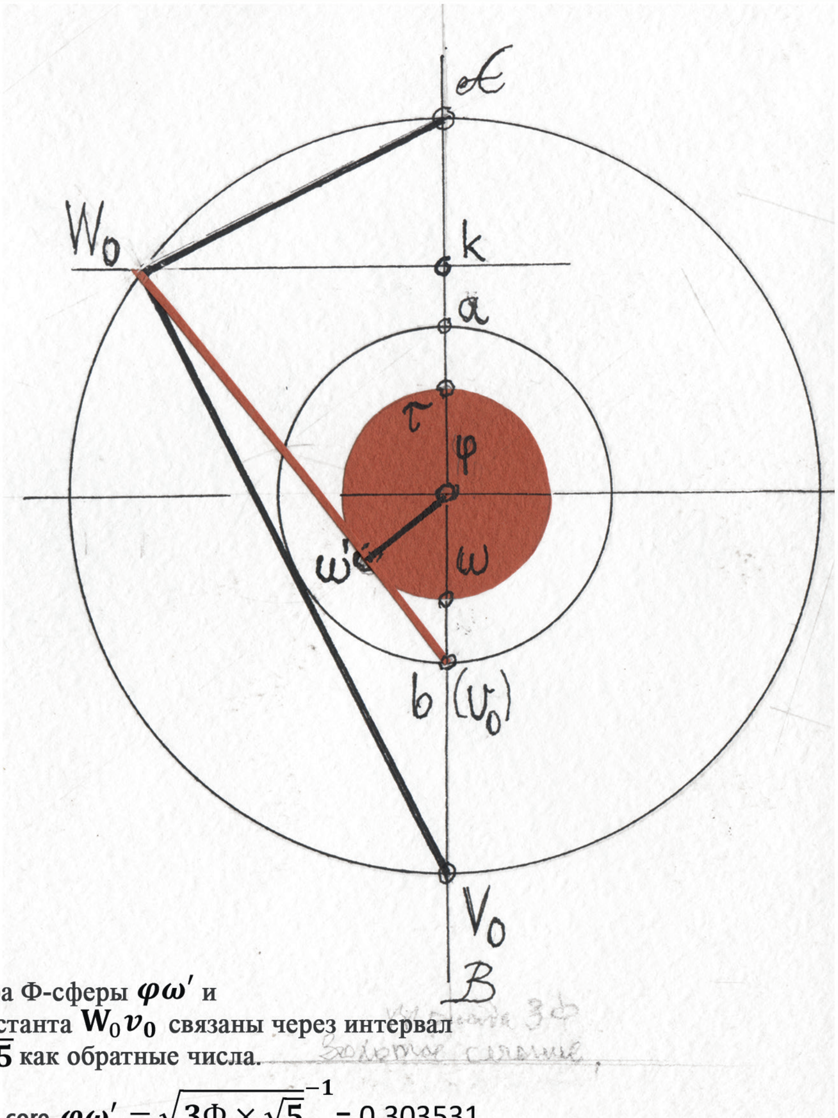


рис. 9.2 **Третья константа:** $W_0\vartheta_0 = \sqrt{\frac{3\Phi}{\sqrt{5}}}$ **= 1.473370.**

fig. 9.2 Third invariable: $W_0\vartheta_0 = \sqrt{\frac{3\Phi}{\sqrt{5}}}$ **= 1.473370.**

Рисунок
Figure
9.2



Радиус ядра Ф-сферы $\boldsymbol{\varphi\omega'}$ и третья константа $\mathbf{W}_0\boldsymbol{v_0}$ связаны через интервал времени $\sqrt{\mathbf{5}}$ как обратные числа.

Radiusof this core $\boldsymbol{\varphi\omega'} = \sqrt{\mathbf{3}\Phi \times \sqrt{\mathbf{5}}}^{\,\mathbf{-1}}$ = 0.303531
are connected through time interval θ = ($\sqrt{\mathbf{5}}$) as inverse numbers.
It is a mathematical fact: having increased the third invariable by a factor of $\sqrt{\mathbf{5}}$,
we find a number reciprocal to core radius $\boldsymbol{\varphi\omega'}$.

Рисунок Figure 9.3

GOLDEN NUMBERS. ORTHO- AND HEXAGONAL SYMMETRIES

13 More than quarter of century ago, having written down the Golden Section equation $\Phi^{\pm 2} = \mathbf{1} \pm \Phi^{\pm 1}$ (with $1 = \omega^0$) in algebraic form

$$\omega^{(\pm 2^{\pm 1})} = \omega^0 + \omega^{+1}, \qquad (17)$$

I have presented it otherwise, as a vector equation, in which numbers $\boldsymbol{\omega}^{\mathbf{n}}$ are expansion modules; vector $\vec{S}$ represents a potency of the Point of origin, vector $\vec{\mathbf{U}}$ – form-building effect of a field comprising the Point of origin: an individual life belongs to the field of life.

$$\vec{\mathbf{R}} = \vec{S} + \vec{\mathbf{U}} \qquad (18)$$

Equation (18) describes interaction of two form-building potencies, ***S*** and ***U***. It maps the duality of life.

Vectors $\overrightarrow{\mathbf{S}_k}$ present potencies of individual lives. These vectors are radially emanated in all directions and are of an equal value: vector module $|\mathbf{S}_k| = \mathbf{1}$. The whole exhibits an image similar to a dandelion blowball.

Vectors $\overrightarrow{\mathbf{U}_k}$, on the contrary, are different-valued. Module $|\mathbf{U}_k| = \boldsymbol{\omega}$ is a variable which depends on angular deviation of its complementary vector $\mathbf{S}_k$ from a biological vertical. As a whole, an aggregate of like-directed vectors $\mathbf{U}_k$ is similar to a dandelion flower-stalk (Fig. 10.2).

The duality principle demands to consider also an alternative version of form-building with shuffling of module roles: the form-building number $\boldsymbol{\omega}$ changes its role of module **U** for that of module **S:** $|\mathbf{S}_k| = \boldsymbol{\omega}$, $|\mathbf{U}_k| = 1$.

Vector $\vec{\mathbf{R}}$ reproduced graphical images on a sheet of paper. These are *the sections* of several basic geometrical forms of wildlife cut along the biological vertical. An apple in which the seed-bud center has coincided with the origin of polar coordinates; outlines of seashell *Pecten* and horseshoe crab carapace; the form of egg of birds-of-pray (eagles, sea eagles, falcons) and ducks (Anatinae); a contour of cranial capsule of mammals, the European skull form, and a symbolical "protoegg" with two planes of symmetry (ab ovo, "all live from the egg") (Fig. 10.3-4). All of this follows a single equation[6]. Eight "square" indicatrixes were built:

four ***S****-symmetries* (with $\mathbf{S}_k$ dominating) and four ***U****-symmetries* (with $\mathbf{U}_k$ dominating);

four ***"plus****-symmetries"* and four ***"minus****-symmetries"*.

The working scheme of vector addition for case **U** is shown in Fig. 10,2.

The model will work, if two conditions are observed, none of which deductable from mathematical rules: 1) *prohibition* on interaction between homogeneous vectors applied to point O_1 (S↔S and **U**↔U); 2) *permission* for interaction of vectors belonging to heterogeneous pairs: $\overline{S_k} \leftrightarrow \overline{U_k}$. It is a literal repetition, in a new situation, of the *prohibition* for interaction A↔ B, a↔**b** and *permission* for interactions (A↔a) ↔ (b↔B), which satisfies the condition of converting the Pythagorean Theorem to the Golden Section. This fact is of utmost importance.

[6] Joseph Shevelev; *The Golden Number and Biosymmetry*; Biology Forum, vol. 87 – 2/3, Perugia, Italy.

14 The second important mathematical fact: G*olden numbers* $\Phi^{\pm 1}$, $\Phi^{2^{\pm 1}}$ are modules of expansion in *orthogonal directions* of "plus-*symmetries***"**. In "minus-*symmetries*" the indicated numbers are not present in ortho- and hexagonal directions. These directions of expansion define other modules, and they also can be named as "golden" owing to their explicit relationship. Such modules are roots of integrity equation $\sum_{n=1}^{\infty} \omega^{(\pm n)} = 1$; they all are numbers ω. We refer them to as the Golden *upper* Φ_w, Golden *lower* Φ_l, Golden *small* Φ_{sm} and Golden *great* Φ_g numbers. They are roots of binary and ternary form-building equations.

1) *Binars:*

number $\omega = \Phi$ is a root of eq. $\omega^{+1} + \omega^{-1} = 1$; $\omega = 1.618034...$ $\omega^{-1} = 0.618034...$

number $\omega = \Phi_u$ is a root of eq. $\omega^{-1} + \omega^{-3} = 1$; $\omega = 1.4655712...$ $\omega^{-1} = 0.6823278...$

number $\omega = \Phi_l$ is a root of eq. $\omega^{+2} + \omega^{+3} = 1$; $\omega = 0.7548777...$ $\omega^{-1} = 1.3247178...$

2) *Ternars:*

number $\omega = \Phi$ is a root of eq. $\omega^{-1} - \omega^{-3} + \omega^{-4} = 1$ $\omega = 1.618034...$ $\omega^{-1} = 0.618034...$

number $\omega = \Phi_u$ is a root of eq. $\omega^{2} + \omega^{3} + \omega^{4} = 1$ $\omega = 1.4655712...$ $\omega^{-1} = 0.6823278...$

number $\omega = \Phi_l$ is a root of eq. $\omega^{3} + \omega^{4} + \omega^{5} = 1$ $\omega = 0.7548777...$ $\omega^{-1} = 1.3247178...$

number $\omega = \Phi_{sm}$ is a root of eq. $\omega^{1} + \omega^{2} + \omega^{3} = 1$ $\omega = 0.5436891...$ $\omega^{-1} = 1.8392864$

number $\omega = \Phi_g$. is a root of eq. $\omega^{4} + \omega^{5} + \omega^{6} = 1$ $\omega = 0.8000950$ $\omega^{-1} = 1.2498515$

Vector **R**, which represents one of the "golden" number $\omega^{(\pm 2^{\pm 1})}$ values, ingeniously outlined from the Point of origin O_1 the basic live nature forms. The numbers of tetrahedron $\sqrt{\Phi}$ (representing the symmetry-of-similarities space) proved to be the expansion modules in regular space division directions.

THE SYMMETRY-OF-SIMILARITIES SPACE AND PERCEPTION OF IMAGES

15 The Second Pythagorean Theorem, in a plane projection, exhibits a circle created by points W and V, where each point is represented by a pair of incommensurable numbers. Their multitude is innumerable. But there are two points on golden sphere Φ, which are unlike all others (Fig. 0.1, 12,1).

A set of points W, V makes, in total, a double golden sphere. The *golden sphere* is *a whole formed from integers,* conjugated in pairs on the principle of incommensurability. Points $\mathbf{W}_\Phi$ and $\mathbf{W}_{\sqrt{\Phi}}$ are basically heterogeneous. They are golden points on a golden sphere: their distances from poles are expressed not in integers, as this takes place in case of points W and V, but through the golden proportion.

$$W_\Phi A / W_\Phi B = \Phi^{+1};\ W_{\sqrt{\Phi}} A / W_\Phi B = \Phi^{1/2}.$$

Projection of points W_Φ and W'_Φ on the diameter of circle AB trichotomizes it in three different ways:

in case $\mathbf{W}_\Phi$, the *Minor golden triad* is built ($\Phi^{-1} + \Phi^{0} + \Phi^{-1} = AB$);

in case $W_{\sqrt{\Phi}}$, the *Great golden triad* is built ($\Phi^{+1} + \Phi^{0} + \Phi^{+1} = AB$).

Point $W_{\sqrt{\Phi}}$ expresses the essence of harmony, inasmuch as it inscribes into the circle the so-called A-rhombus, symmetry-of-similarities space, which is closed, finite and, moreover, infinitely descending in its own depth (Fig. 12.1,2,4). An element of this structure is the triangle of Price (Fig. 12.4). Three sides of this triangle are interconnected as numbers $\sqrt{\Phi}^{-1}$, 1, $\sqrt{\Phi}^{+1}$. The triangle of Price has created an A-rhombus structure by connecting all points to the rhythm of $\sqrt{\Phi}$.

In the following section we shall pass from the plain symmetry-of-similarities space to a 3D-space of golden tetrahedrons originating from the same triangle $\sqrt{\Phi}$. But the fundamental law of harmony, the law of structurization of natural systems, can be elementary and precisely expressed by figures *on a plane*, in terms of elementary geometry, using the compasses and ruler. It is easier. And there is no other way, however paradoxical it might sound.

16 Whatsoever is many-dimensional, complicated and boggling the mind, – all of this can (and shall!) be returned to its simple source, i.e. abstracted to one number and one picture.[7] Exactly so behaved the Nature when creating biological mechanisms of perception: sight, hearing, sense of smell, taste, tactile sensations. All of them are arranged so that symbols of the real world originate in live systems on the “perception horizon”, i.e. on *a boundary surface* (interface). On one side of the boundary surface ("the perception diaphragm") is the external world, “non-ego”. On another side – "I", integration system, the spiritual world subject to the laws of harmony.

The eye perceives light and color images of the external world, projecting them with eye-lens on retina receptors. Retina is a facial layer of neurons covering the bottom of eyeball.

Hearing organs receive acoustic waves acting upon the eardrum (a surface).

Organs of smell and taste use dendrites – the transmitters arranged on surfaces of nasal cavity (smell) and tongue (taste) – to perceive signals.

Touch is an effect resulting from contact between surfaces of the external world bodies and skin covers, finger tips, the hairs incorporated in skin covers.

Integral interpretation and handling of the information gained from all types of detectors happens in the right and left hemispheres of the cortex cerebri, – in a binary structure, peckled with cerebral gyri, i.e. in ***surface layers*** of the brain cortex. Thus, the biostructures responsible for information transfer (just as the Golden space) discover a "diaphragm" dividing the world of ego into two implicitly paradoxical areas. On the one side is the nature organized under the laws of harmony, "conducted", experimentally accessible, but inexplicable regarding something most important. On the other side is an "unfathomable" zone: the mysterious world of perception, spirit and intuition. The meeting of two worlds, "the visible” and “the invisible", generates ***symbols.*** These, upon being coded, assume a finished graphical form given by human sense and reason and are *rendered by human arm.* The hieroglyphs, inter alia letters, numbers, notes, formulas, pictures and drawings, arose just on ***the surfaces*** similar to those which the nature uses to separate and connect the inner and exterior worlds, on a horizon of incognizable: differentiated images of the intelligible world coded by light, color, lines, plastics, patterns and ratios.

Let us return to the golden points of sphere, $\mathbf{W}_\Phi$ and $\mathbf{W}_{\sqrt{\Phi}}$. Projection of points W_Φ and W'_Φ on the diameter of circle AB trichotomizes it in unique ratios (Fig. 01, 12.1). Thus we have built *the Minor golden triad* ($\Phi^{-1} + \Phi^0 + \Phi^{-1} = AB$)

The position of point $\mathbf{W}_\Phi$ ($W_\Phi A$ / $W_\Phi B = \Phi^{+1}/1$) is crucial for inscribing double square $W_\Phi W'_\Phi$ into the Φ-sphere drawing, which is of fundamental importance in proportions of the Mediterranean architecture[8].

The position of point $\mathbf{W}_{\sqrt{\Phi}}$ is defined by its distance from poles A, B showing the relationship $\Phi^{+1/2}/1$. Point $W_{\sqrt{\Phi}}$ and its twins $W'_{\sqrt{\Phi}}$ are located so that the projection of these points on the diameter of circle AB trichotomizes the letter. Thus we have built the unique

Great golden triad ($\Phi^{+1} + \Phi^0 + \Phi^{+1} = AB$)

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[7] About geometrical similarity in visual perception and formation of reason, see: И. Шевелев. Золотое пространство. Промдизайн-М. Кострома, 2006 (J. Shevelev; *The Golden Space*; Promdesign-M Publishers; Kostroma, 2006).

[8] See more in detail: И. Шевелев. Искусство архитектуры. В кн. «Основы гармонии». М., Луч, 2009, с. 14-32. (J. Shevelev. *The Art of Architecture*. Lib.: *The Fundamentals of Harmony*; Moscow, “Luch” Publishers, 2009, p. 14-32).

The Great golden triad via the golden proportion integrates parts into a whole not fourfold, as the *Minor* triad or the *Ascendant* triad, but eight times.

The Great golden triad had played an outstanding role in the history of the Russian medieval architecture[9].

THE SECOND PYTHAGOREAN THEOREM (GOLDEN SPHERE) AND ELLIPSOID $\sqrt{\Phi}$

17 In mathematics, a sphere is considered to be an ellipse which axes are equal (М:Б = 1/1 = 1) and two focal points coincide. The Second Pythagorean Theorem equation $\equiv$ symmetry-of-pairs equation, ergo both of them represent a sphere. A drawing of the "optimistic **solar** ellipse" by Prof. Georgy Darvas (in my terms, the same as the "**Golden** ellipse") awoke desire to fathom what connections exist between ellipses, all of them, and ***8 biosymmetries*** built by quadratic equation of integrity $\omega^{\pm 2^{\pm 1}} + \omega = 1$, when it is considered as a vector equation. The Second Pythagorean Theorem equation $\equiv$ symmetry-of-pairs equation, as evident from the above,[10] has bared the units of natural geometry hidden in a circle (hence also in a sphere) and generating each other. In the "Units of natural geometry" (Figs. 2.2, 3.3, 8, 9) it is shown[11], how in the "Point of Origin", which is a sphere (a geometrical image of the symmetry-of-pairs equation, number Ф), are merged and generate each other numbers Ф, 1, $\sqrt{5}$ and 1, 2, $\sqrt{3}$. The "ellipse evolution" has linked these constants quite as closely: discrete transformation of a circle inscribed in a square – into an ellipse inscribed in similar rectangles and also circumscribing them (Table 5).

Table 5. "Evolution" of the golden structure constituted of ellipse parameters Б/М=1, $\sqrt{\Phi}$, $\sqrt{5}/\sqrt{3}$, $\sqrt{2}$; Ф ; $\sqrt{3}$, 2 and $\sqrt{5}$ (where **1** = $\Phi^{+1} - \Phi^{-1}$; **2** = $\Phi^{+2} - \Phi^{-1} = \Phi^{+1} + \Phi^{-2}$; **3** = $\Phi^{+2} + \Phi^{-2}$) and number $\sqrt{2}$

Ellipse No.	Axes ratio, Б/М	Ellipse focal distance, F	Eccentricity ratio, Б/F	Rectangle aspect ratio, $m_{(inscr)}/M_{(circumscr)}$
8	$\sqrt{5}/1$ 2,236	2	$\sqrt{5}/2$	$\sqrt{2}$
7	2/1 2,000	$\sqrt{3}$	$2/\sqrt{3}$	$\sqrt{2}$
6	$\sqrt{3}/1$ 1,732	$\sqrt{2}$	$\sqrt{3}/\sqrt{2}$	$\sqrt{2}$
5	Ф/1 1,618	$\sqrt{\Phi}$	$\sqrt{\Phi}$	$\sqrt{2}$
4	$\sqrt{2}/1$ 1,414	1	$\sqrt{2}/1$	$\sqrt{2}$
3 (proto-egg)	$\sqrt{5}/\sqrt{3}$ 1,291/1 1,291	$\sqrt{2}/\sqrt{3}$ 0,8165	$\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{2}}$	$\sqrt{2}$
2	$\sqrt{\Phi}/1$ 1,272	$\sqrt{\Phi}^{-1}$	Ф/1	$\sqrt{2}$
(circle)	1/1 1,000	0	F → Б	non-existent

The model (Fig. 11.2) has shown: the ideal form, an ellipse (geometrical figure), and the live form (a curve reproduced by the vector equation of integrity) have not coincided.

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[9] Ib., p. 106-139.

[10] See Lib.: И. Шевелев. Гармония в зеркале геометрии. 2013. pp. 17-18.

[11] See: И. Шевелев. Константы естественной геометрии. On website: ishevelev.ru, 2015.

Closed curve No.3 built by linear equation $\overrightarrow{\omega^{-1}} = \vec{\omega} + \vec{1}$ is strictly doubled by a curve built by quadratic equation $\overrightarrow{\omega^{\frac{1}{2}}} = \vec{\omega} + \vec{1}$. In terms of the "protoegg" ellipse parameters, it is a pseudo-ellipse since its parameters are set differently:

1) in a *linear* equation of integrity the focal distance is a *constant*: $O_1O_2 = 1$, whereas the radii are variable reciprocals: m $O_1 = \omega$ and m $O_2 = \omega^{-1}$
2) in a *quadratic* equation the focal distance is a *variable*: $O_1O_2 = \omega$, whereas the radii are of different kinds: one of them is constant m $O_2 = 1$, the other – function of variable ω, m $O_1 = \omega^{\frac{1}{2}}$.

Commensurability of a "live" ellipse: the major and minor axes, Б/М = $\sqrt{5}/\sqrt{3}$
Superimposing the "live" ($\sqrt{5}/\sqrt{3}$) and canonic [($\sqrt{5} \times \sqrt{3}$)/1] ellipses (Fig. 11.2) brings about the following results:

the extremes coincided, the curve divaricated. The live ellipse ("proto-egg") became more springy, "fleshy", plumpy. It is the golden ellipse. Its major axis is divided by focal distance in the minor golden triad ratio (Φ^{-1}, 1, Φ^{-1}).

The greatest vertical deviation of the "live ellipse" curve from that of the classical ellipse built with same parameters М:Б amounted to +1/69.
In a classical ellipse (М:Б = $\sqrt{\Phi}$/1) *inverse numbers* would be *axis* Б = $\sqrt{\Phi}$ and *focal length* $F_1F_2 = \sqrt{\Phi}^{-1}$. *In a "live" ellipse* (М:Б = $\sqrt{5}/\sqrt{3}$) *inverse numbers* are *axis* М = $\sqrt{3}$ and *focal length* $O_1O_2 = \sqrt{3}^{-1}$.

Any ellipse can be inscribed into rectangle М:Б, where М and Б are axes of the ellipse, whereupon we can inscribe into this ellipse rectangle m:b, similar to rectangle М:Б. As is clear from Fig. 2, the ratio of small sides of the inscribed and circumscribed rectangles *in any ellipse* is the same (**m: M = 1 : $\sqrt{2}$**). I have not met constant $\sqrt{2}$ among the ellipse parameters in reference books. At the same time in genetics and physics (in *natural* geometry), as well as in art, bisection, doubling, $\sqrt{2}$, geometrical similarity and inverse numbers are fundamental. Showing up the $\sqrt{2}$-based similarity of inscribed and circumscribed rectangles of ellipse is of utmost importance in the problem of form-building. The ratio of similitude $\sqrt{2}$ is one of the latent but essential parameters of the ellipse.

18 Intriguing storyline is that for a circle as a generate form of ellipse, ***the position of focal points in poles*** A, B ***is impossible***. But *the Pythagorean circle (symmetry-of-pairs equation) is built not by radius, as is customary when plotting a circle, but* ***from two poles,*** *in the same way as in the construction of any ellipse*. The eccentricity is set by $F_1F_2 <$ Б. In the golden sphere, where algorithms of metamorphoses are contracted, ***the disposition of focal points in poles A, B, on the contrary, is a necessity***: just the polar location of two centers has created the Second Pythagorean Theorem and converted it to the Golden Section $\equiv$ the symmetry-of-pairs equation, the algorithm of life. Focal points (A and B) *have fallen outside the limits defined by the ellipse equations.* When initially coincident points F_1, F_2 have attained opposite boundaries of the ellipse (FF = Б = 1), the ellipse has disappeared.

The Pythagorean Theorem assumes that the circle is double; there are two coincident circles. They are nested into each other since are built by two incommensurable pairs of numbers, N/1 and θ/1, i.e. are created by points of the surface which distances to polar focal points are incommensurable. Thereby two ***complementary*** circles (spheres), easily penetrating each other, enter each other, creating the third sphere, a sphere-the-whole, not colliding at any point, and become parts of a new ***whole,*** which is a structure of a higher complexity level, everyone of components keeping its integrity, separate identity, "personality". This metamorphosis also is transformation of the Pythagorean equation to the Golden Section.

It is significant that the event "*vanishing*" scenario mathematically is inverse *to the formation* scenario. Transformation of the Pythagorean Theorem to the symmetry-of-pairs equation occurs instantly. This is rearrangement of the equation describing numberless points of a spherical surface, into the equation describing only *interaction of its two poles*. The sphere-ellipse, number *Unit* (М:Б = 1:1 = 1), incomprehensible to human reason by the Galilei definition, – has passed in a space of metamorphoses with "an imaginary ellipsoid" (М:Б = 0:N) as its limit, where $1 > N \to \infty$. Both events: metamorphosis of the Pythagorean equation to the Golden Section and transformation of a common ellipsoid to an imaginary ellipsoid ("life" $\leftrightarrow$ "non-existence") are presented by the same algorithm. *Here, 0 and 1 are connected by a closed cycle of transformations.* This is divarication of the integral whole: juxtaposition and separation of points F_1, F_2 in the equation of ellipse or conjunction and separation of spherical surfaces with diameters $\sqrt{5}$, 1, $\sqrt{2}$ and $\sqrt{\frac{3\Phi}{\sqrt{5}}}$, which are created by transformation of the Pythagorean equation to the Golden Section algorithm or, in other words, to the Symmetry-of-pairs algorithm contracted to the "Point of Origin" sphere.

TETRAHEDRON $\sqrt{\Phi}$ AND SPACE DIVISION

19 The triangle belongs to a plane. Emergency of the fourth point creates a 3D space. The simplest one among five *regular* Platonic bodies has four *equilateral triangular* faces. It is impossible to tightly fill out (to pack, as crystallographers have it) the unlimited, continuous 3D space with *one Plato tetrahedron.* It is necessary to alternate tetrahedrons with octahedrons in the ratio of 2:1. The reason is that a "*regular*" tetrahedron does not contain ***the right angle*** and, therefore, a sphere, a circle (number **π)**, i.e. the idea of movement, expansion.

The *Natural* Unit, a module of real space, should build angles $\frac{\pi}{3}$ and $\frac{\pi}{2}$ (crystals and waves), as well as angles $\frac{\pi}{5}$ (wildlife). It has also to conceal in itself, in a contracted form, the primary symmetry laws. Such a structure possessing physical and psychobiological sense is ***tetrahedron*** $\sqrt{\Phi}$, the 3D module of the symmetry-of-similarities space (SSS). Its linear constituent, an edge of tetrahedron $\sqrt{\Phi}$, is a variable. Six edges of a tetrahedron represent number Φ^n, where n = 0, 1, $\pm\frac{1}{2}$. Four interfacial angles are $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{5}$, and an angle equal to $\frac{1}{2}$ of the angle of intramolecular bond in a water molecule, approximately 104^{o} (Figs. 11,2 and 12.4,5).

Tight packing of space with a unique tetrahedron $\sqrt{\Phi}$ is described in detail in my work "Other space"[12].

20 It is known that the problem of space division by one regular (Platonic) tetrahedron has no solution. Tetrahedron $\sqrt{\Phi}$ solves this problem when treating space as not a static, but dynamic object. The tetrahedron changes edge length $\sqrt{\Phi}^{\pm n}$, where n = 0, 1, thereby changing

[12]See: И. Шевелев. Другое пространство. Авенир-Дизайн., Кострома, 2010 (J. Shevelev; *A Different Space*; “Avenir-Design” Publishers, Kostroma, 2010); also: И. Шевелев. Гармония в зеркале геометрии. ДиАр., Кострома. 2013 (J. Shevelev; *The Harmony in a Mirror of Geometry*; “DiAr” Publishers, Kostroma, 2013).

its form**,** but **its volume remains unaltered.** The body of tetrahedron **ω** = $\sqrt{\Phi}$ "breathes". Owing to this feature, the dynamic tetrahedron, ***solo***, is capable to fill out 3D space *absolutely tight*[13] (Figs. 12, 13, 14). Unit **ω =** $\sqrt{\Phi}^{\pm 1}$ *and rhythm of changing the edge length* are defined by a single number $\sqrt{\Phi}$**.** Tetrahedron $\sqrt{\Phi}$ fills in the unlimited continuous space using two independent techniques: as a *minor* or *major* structure***.*** Alternatively, it divides space on a binary basis, combining the *minor* and *major* tetrahedrons layer-by-layer.

The rule of tight packing the space with the golden tetrahedrons is more sophisticated than that of Plato. Three equal-sized tetrahedrons $\sqrt{\Phi}$ are packed into a regular trihedral prism; there are two alternatives of arrangement. The minor prism is composed of tetrahedrons B, C, B, the major prism – of tetrahedrons A, D, A. The tipped tetrahedrons are equal. Tetrahedron B ↑ is identical to B ↓, tetrahedron A ↑ is identical to A ↓. Space intervals between identical tetrahedrons are filled in with the third type tetrahedrons (C and D), see Fig. 12.5, 13, 14.

Tetrahedrons are linked up at an edge of the equilateral "golden" triangle. Its sides are 1, Φ (in the case "major") or 1, $\sqrt{\Phi}$ (in the case "minor"). 1) If the twins touch one another at *the left* edges of an equilateral side, the tetrahedron enclosed between them is *left-rotating* (in the "minor" space it is tetrahedron $C_{(-)}$, in the "major" space – tetrahedron $D_{(-)}$). 2) If the *right* edges of equilateral sides adjoin, the space between the twin tetrahedrons is a *right-rotating* tetrahedron (minor-C $_{(+)}$, or major - D $_{(+)}$).

Tetrahedrons A, B have a bilateral symmetry plane. Tetrahedrons C, D do not possess bilateral symmetry and consequently can build *left-rotating* and *right-rotating* spirals (Figs. 14, 1a, 15.1-2).

In a space of alternating "minor" and "major" layers it is necessary to select a hexagonal prism: the block of *thirty six* tetrahedrons. *Twelve* of them (six "minor" and six "major" tetrahedrons) constitute the core of this block: the symmetry-of-similarities space module "A-rhombus"[14] (Fig. 13, 1). Each of twelve "A-rhombus" tetrahedrons can be broken up into two tetrahedrons, A and B. This decomposition can be continued ad infinitum. The space of each tetrahedron is diving in its own depth. This is a chain of hierarchies integrated by rhythm $\sqrt{\Phi}$, directed toward both infinitesimal and infinitely great values. As a whole, the **"A-rhomb" structures are two similar, tip-to-tip inverted and interleaved symmetry-of-similarities spaces.**

---

[13] A rectangular trihedral prism, which base is an equilateral triangle, can be cut into three equal-sized tetrahedrons at any breakdown ratio of its vertical face. Two of them are identical, have a plane of symmetry, and are tip-to-tip inverted (so that their bases are arranged at the top and at the bottom). The third tetrahedron has no plane of symmetry. It fills remaining space. As all three tetrahedrons are of the same height **h**, and the volume of trihedral prism **V** = **F** $\times \frac{1}{3}$ h, they all have *equal volume.*

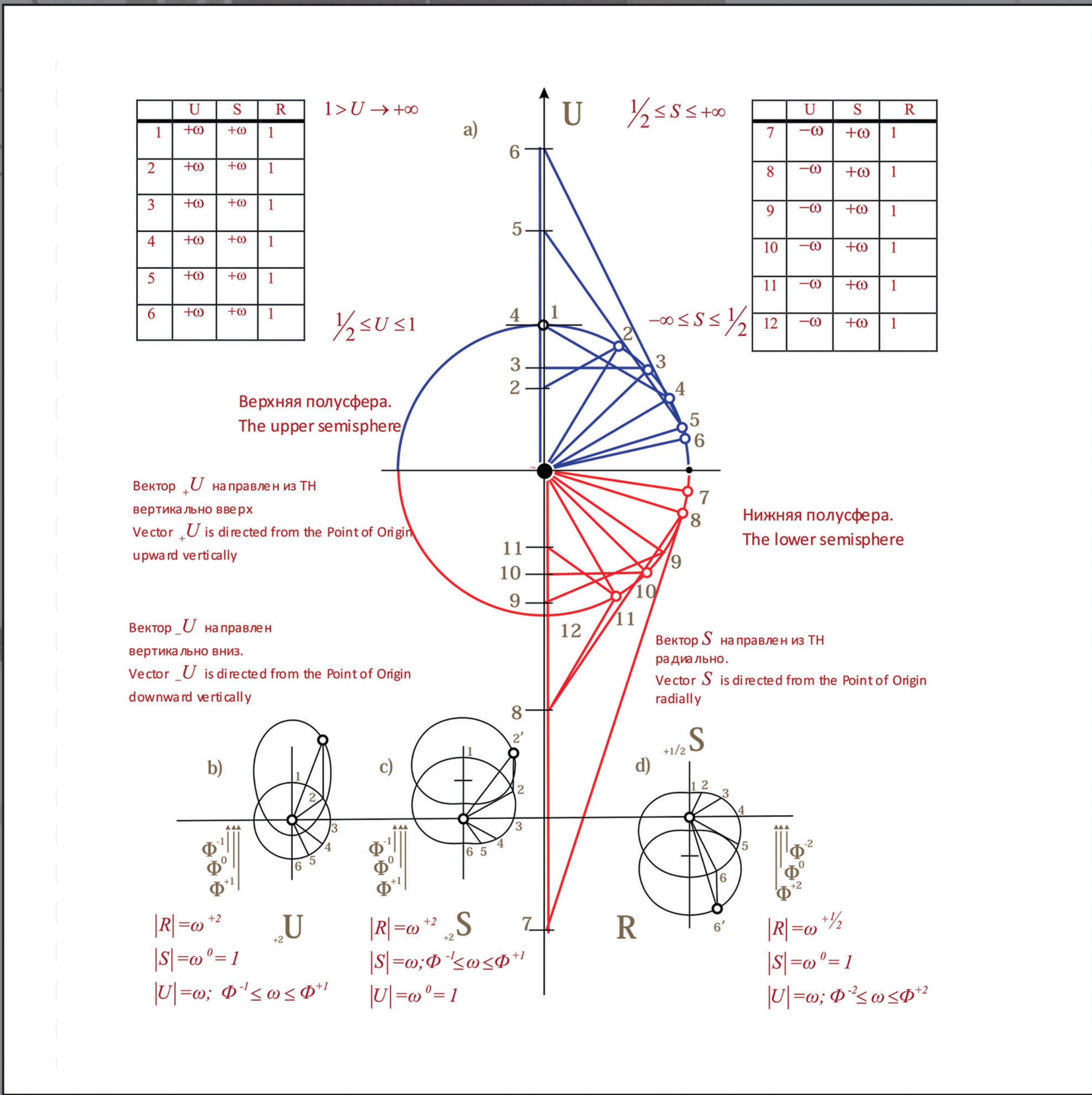
[14] See: И. Шевелев. Другое пространство. Авенир-Дизайн., Кострома. 2010 (J. Shevelev; *A Different Space*; "Avenir-Design" Publishers, Kostroma, 2010).

ИЛЛЮСТРАЦИИ К

II ЧАСТИ

ILLUSTRATIONS

PART II



	U	S	R
1	+ω	+ω	1
2	+ω	+ω	1
3	+ω	+ω	1
4	+ω	+ω	1
5	+ω	+ω	1
6	+ω	+ω	1

	U	S	R
7	−ω	+ω	1
8	−ω	+ω	1
9	−ω	+ω	1
10	−ω	+ω	1
11	−ω	+ω	1
12	−ω	+ω	1

Рисунок
Figure 10.1

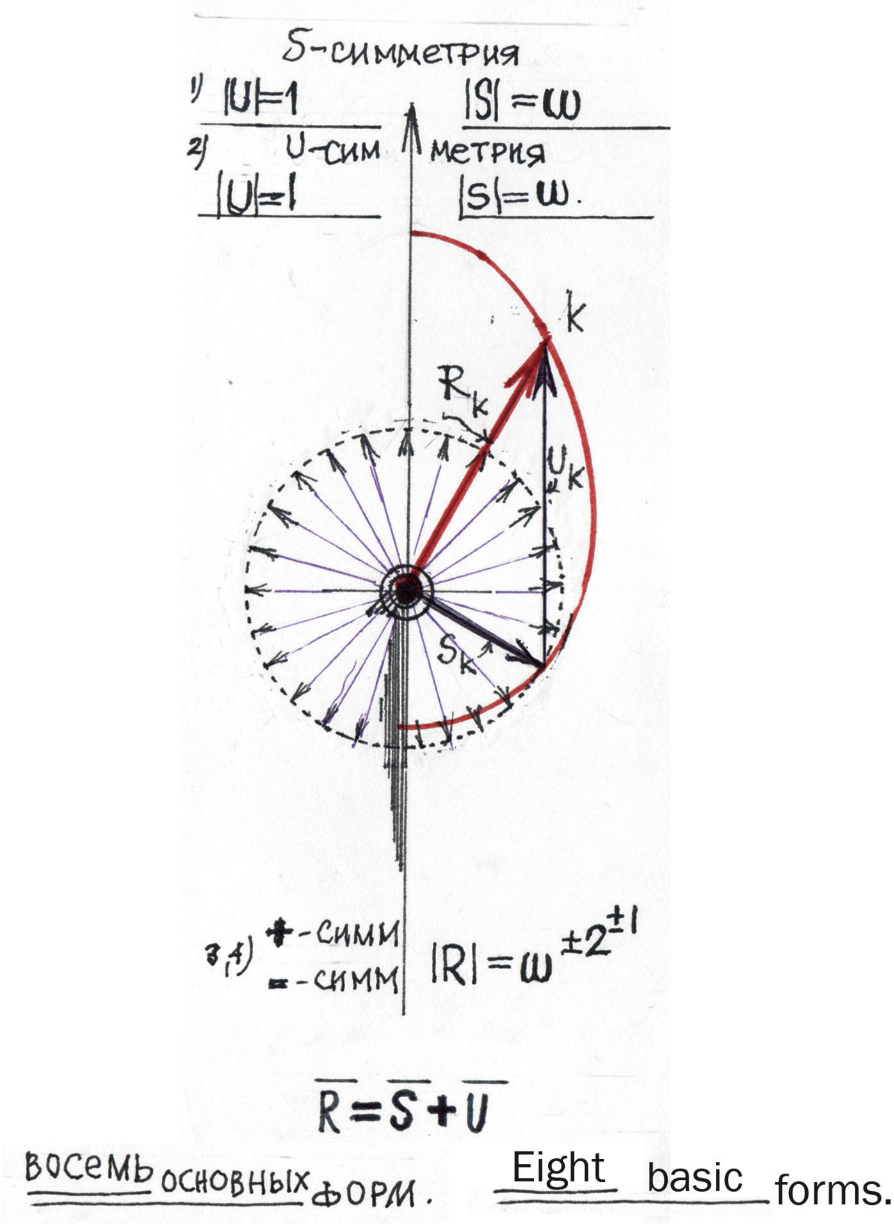


Рисунок
Figure 10.2

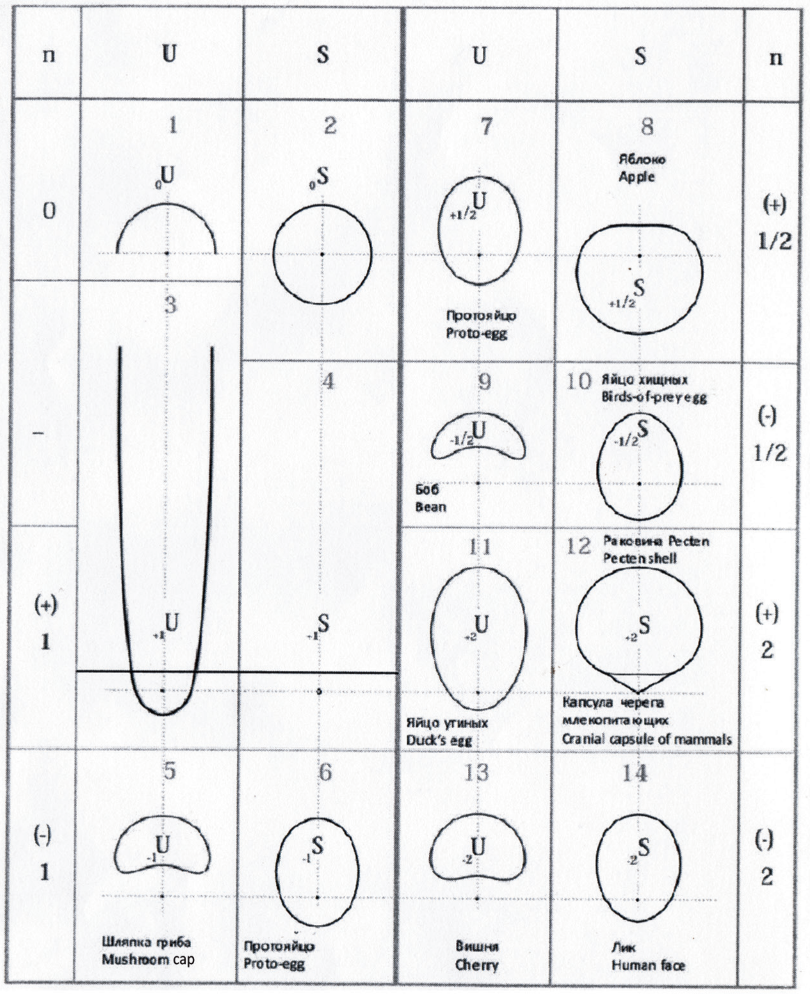


Рис. 38. ***Векторный треугольник $\bar{S},\bar{U},\bar{R}$ и вероятные модели U-, S- пространства***

Слева : 1, 3, 4/ пред бытие: $n = 0; n = +1$; 5, 6/ первообразы замкнутого пространства: $n = -1$.

Справа: 7 – 14/ Биосимметрии $\pm$(U, S), воспроизводящие образы, адекватные основополагающим формам живой природы: $n = \pm 2^{\pm 1}$.

Fig. 38. ***Vector triangle , $\bar{S},\bar{U},\bar{R}$, and possible models of the U-, S-space.***

At left: 1, 3, 4/ preexistence: $n = 0; n = +1$; 5, 6/ the prototypes of closed space: $n = -1$.

At right: 7 to 14/ the generative images adequate to basic forms of wildlife. $n = \pm 2^{\pm 1}$.

Рисунок
Figure 10.3

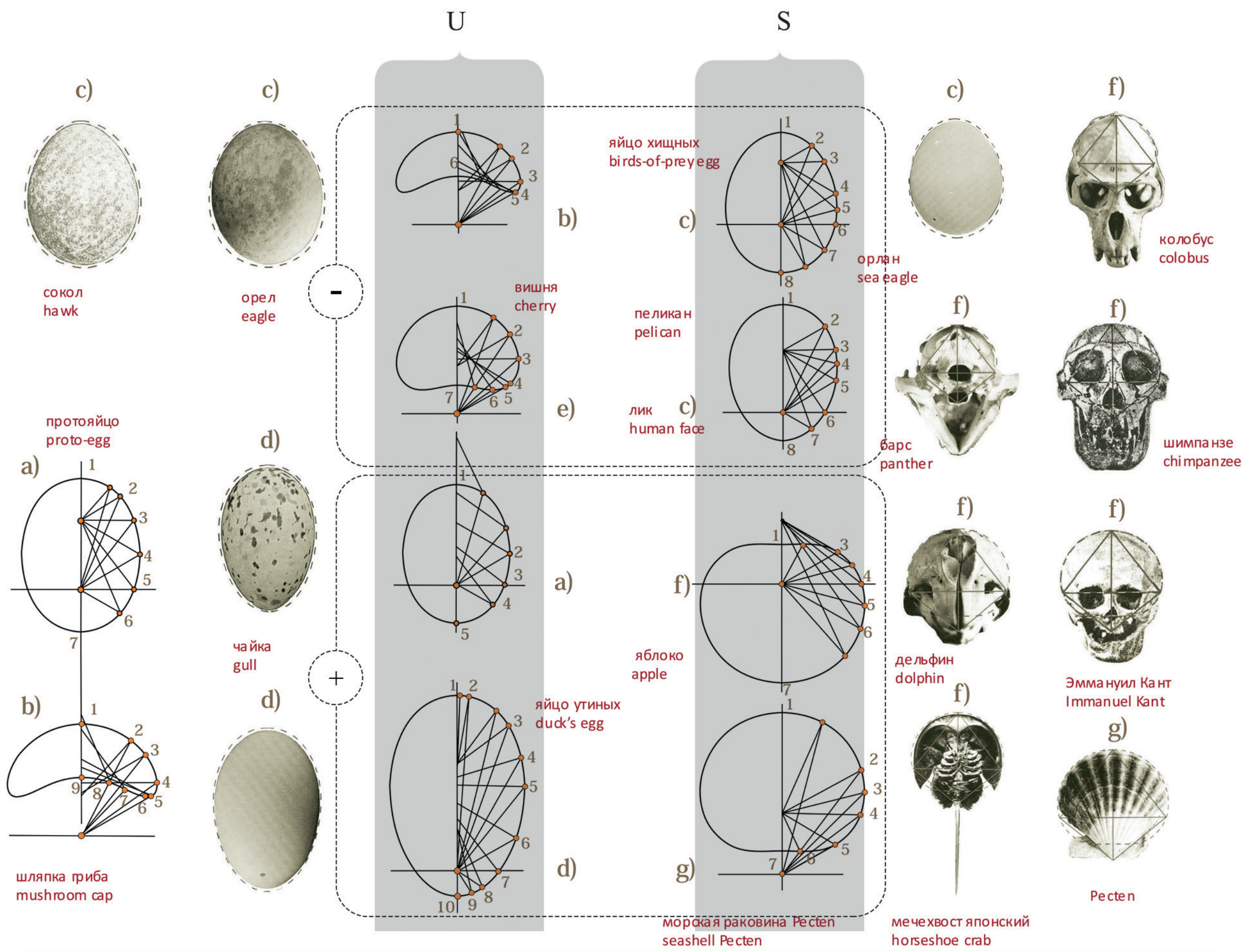


Рис. 39. ***Векторное уравнение $\vec{S},\vec{U},\vec{R}$ и формы живой природы***

Fig. 39. ***Vector equation $\vec{S},\vec{U},\vec{R}$ and the wildlife forms***

Рисунок
Figure 10.4

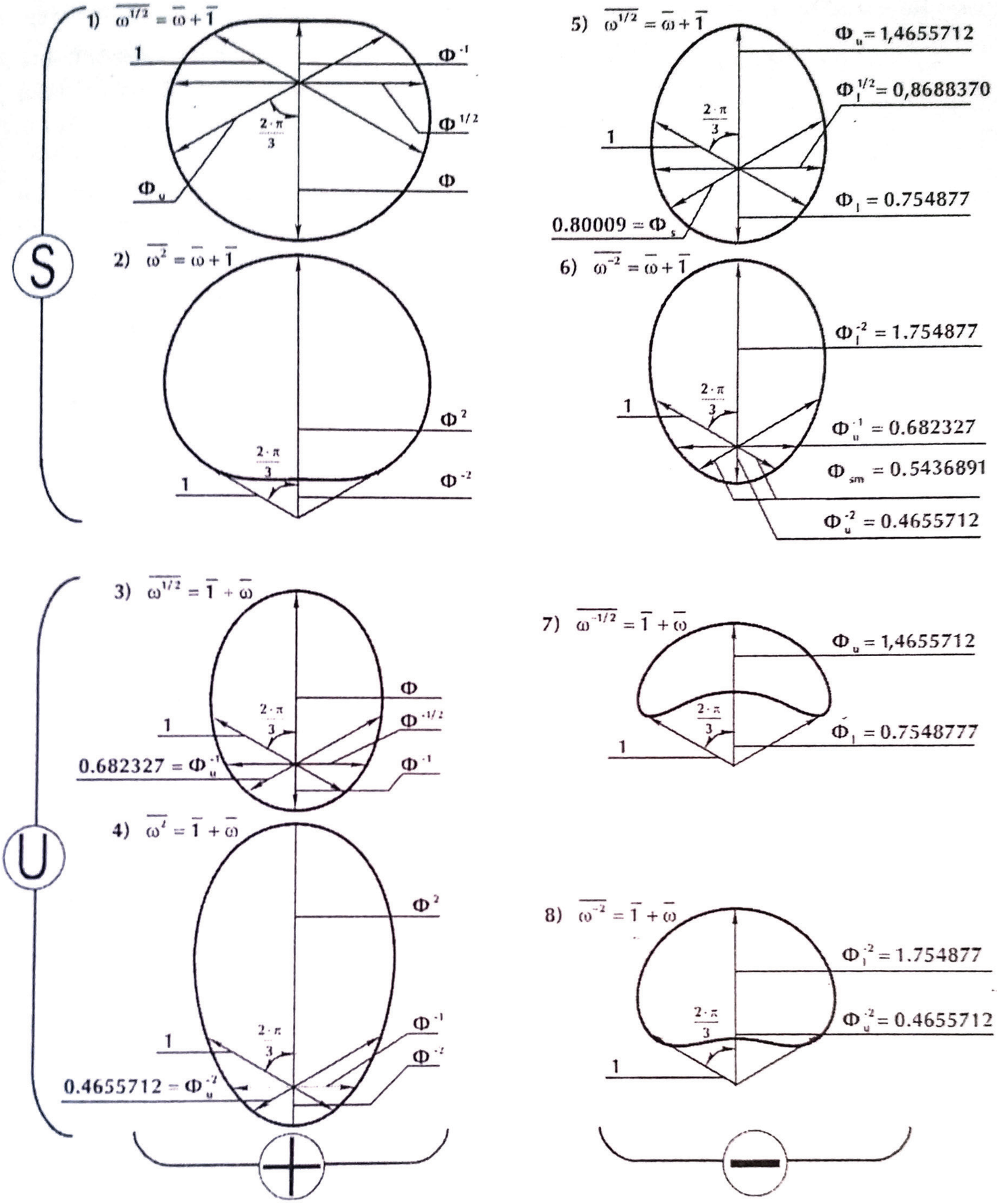
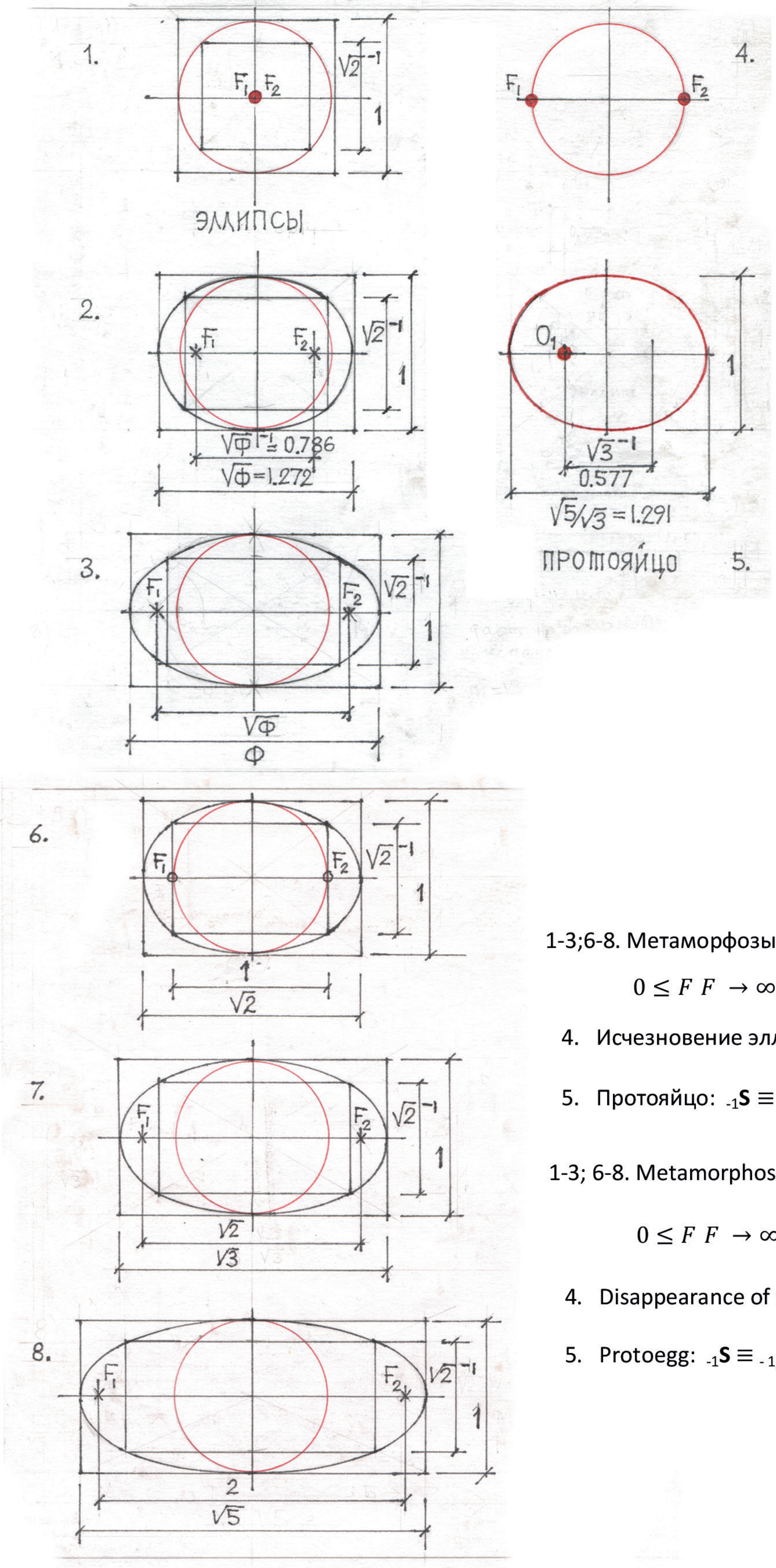


Рис. 10.5 Восемь основополагающих +,-,U, S биосимметрий

Eight fundamental forms in biosymmetry: +,-,U, S



1-3;6-8. Метаморфозы эллипса

$$0 \leq F\ F \to \infty;\ {}^{Б}/_{М} \to \infty$$

4. Исчезновение эллипса при $F_1\ F_2$ = Б.

5. Протояйцо: ${}_{-1}\mathbf{S} \equiv {}_{-1/2}\mathbf{S}$

1-3; 6-8. Metamorphoses of the ellipse

$$0 \leq F\ F \to \infty;\ {}^{Б}/_{М} \to \infty$$

4. Disappearance of the ellipse when $F_1\ F_2$ = Б.

5. Protoegg: ${}_{-1}\mathbf{S} \equiv {}_{-1/2}\mathbf{S}$

Рисунок Figure 11.1

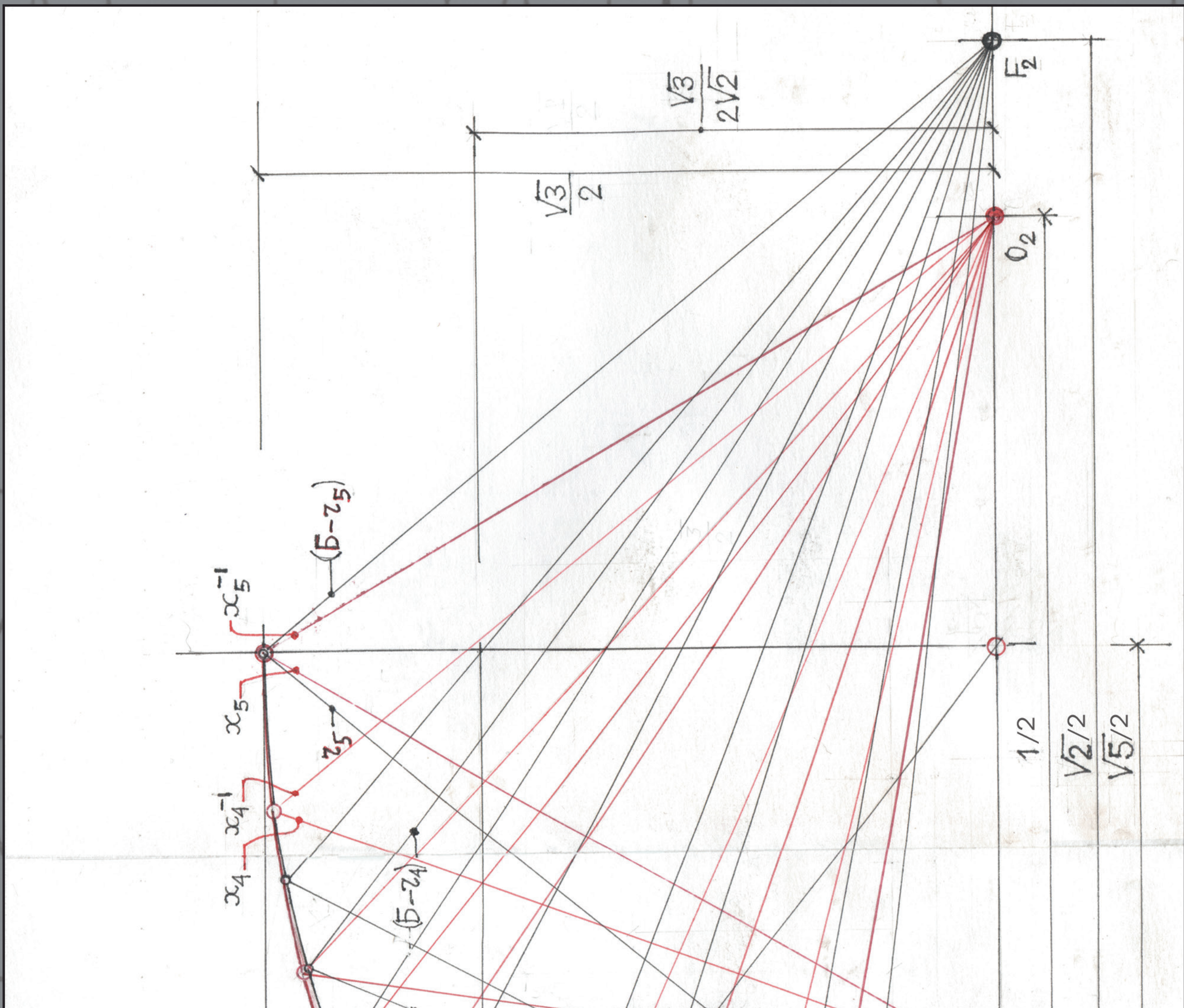


Рисунок
Figure 11.2

Рис. **ПРОСТРАНСТВО СИММЕТРИИ ПОДОБИЙ**

Symmetry-of-similarities space

Непрерывное Подобие

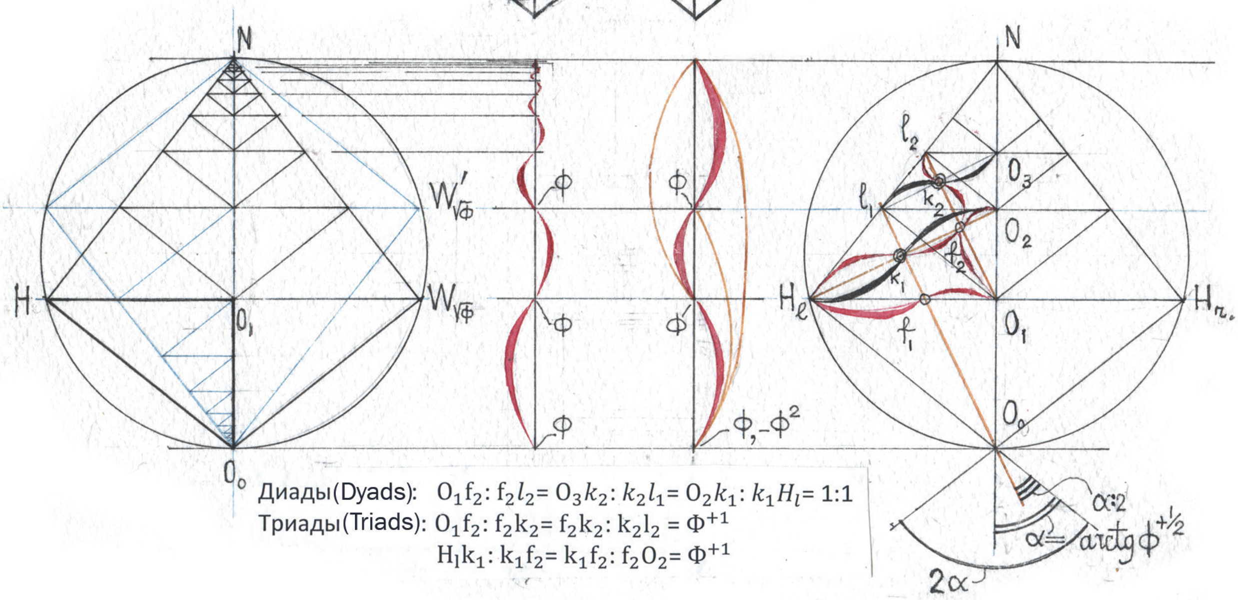
Continuous similarity

Великая золотая Триада

Great Golden Triad

Дихотомия отрезков и углов
Диагональные золотые триады
Внутримолекулярная связь в молекуле воды

Dichotomy of line segments and angles
Intramolecular bond in a water molecule

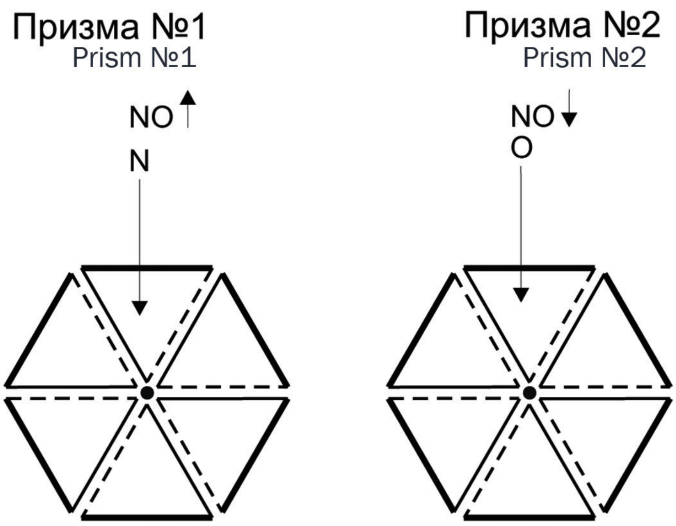


Диады(Dyads): $O_1f_2 : f_2l_2 = O_3k_2 : k_2l_1 = O_2k_1 : k_1H_l = 1:1$
Триады(Triads): $O_1f_2 : f_2k_2 = f_2k_2 : k_2l_2 = \Phi^{+1}$
$H_lk_1 : k_1f_2 = k_1f_2 : f_2O_2 = \Phi^{+1}$

Рисунок Figure 12.1

Правильное соединение граней одноименных и разноименных призм
Correct blocking of tetrahedrons

Соединяются одноименные призмы		**Cognominal prisms**			
		$1\leftrightarrow 1$		$2\leftrightarrow 2$	
		1	2	3	4
		$L_1\leftrightarrow T_1$	$L_1\leftrightarrow R_1$	$L_2\leftrightarrow R_2$	$R_2\leftrightarrow T_2$
Соединяются разноименные призмы		**Heteronymous prisms**			
	5	6	7	8	9
	$L_2\leftrightarrow R_1$	$R_2\leftrightarrow L_1$	$T_1\leftrightarrow T_2$	$T_2\leftrightarrow R_1$	$L_2\leftrightarrow T_1$



Условные обозначения Legend

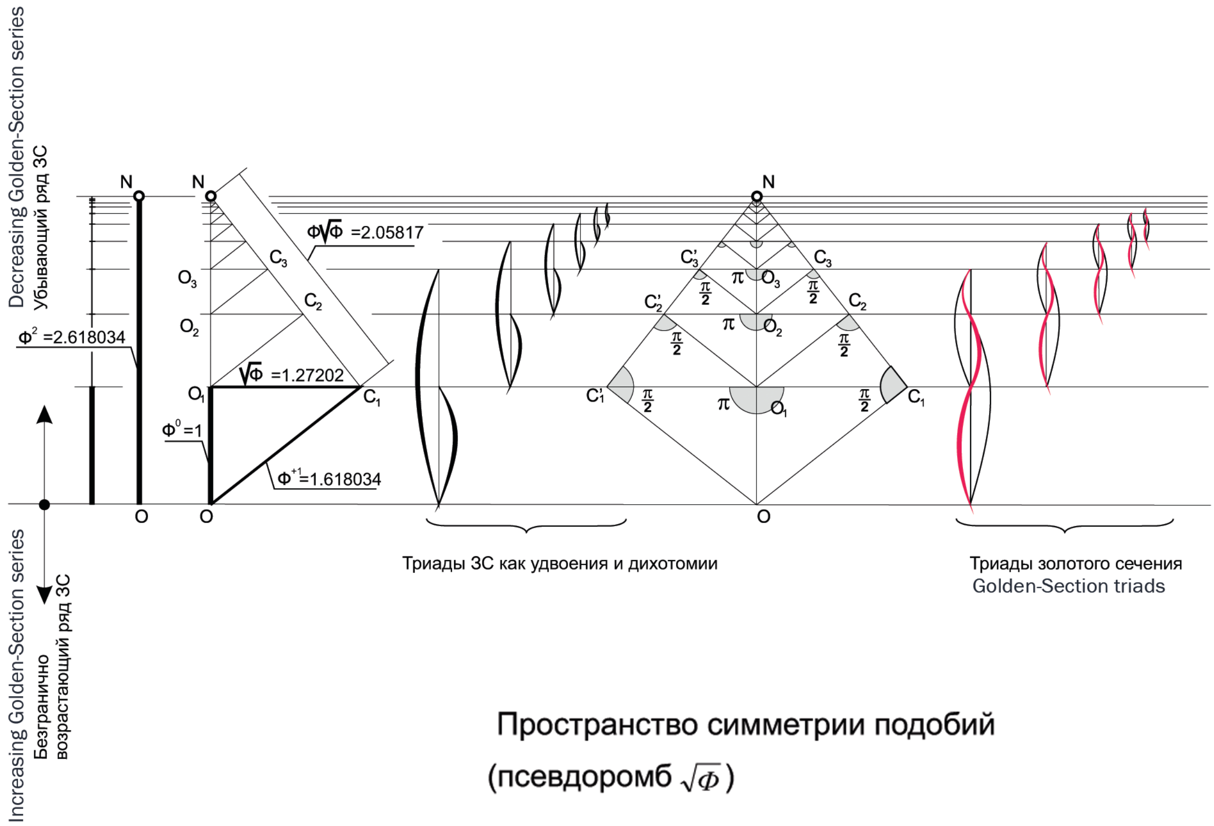
T — Тыл Rear

L — Левый Left

R - - - Правый Right

Рисунок Figure 12.2

равильное соединение граней одноименных и разноим
Соединяются одноименные призмы
1↔1
1 2 3
$L_1\leftrightarrow T_1$ $L_1\leftrightarrow R_1$ $L_2\leftrightarrow$
Условные обозначения



Пространство симметрии подобий (псевдоромб $\sqrt{\Phi}$)

Зеркальная симметрия преобразует убывающий ряд ЗС в пространство симметрии подобий (*ПСП*). Это пространство прямого угла $\pi/2$, π и связей $\Phi^{\frac{1}{2}}$. Назовем эту фигуру « $\sqrt{\Phi}$ -ромб». Треугольник Прайса принимает в нем 8 ориентаций.

Symmetry-of-similarities space

Symmetry-of-similarities space (SSS) is a space dominated by right angle $\pi/2$, π and links $\Phi^{\frac{1}{2}}$. Let us name this figure, created by the Price triangle assuming 8 orientations, "A-rhombus"

Рисунок
Figure 12.3

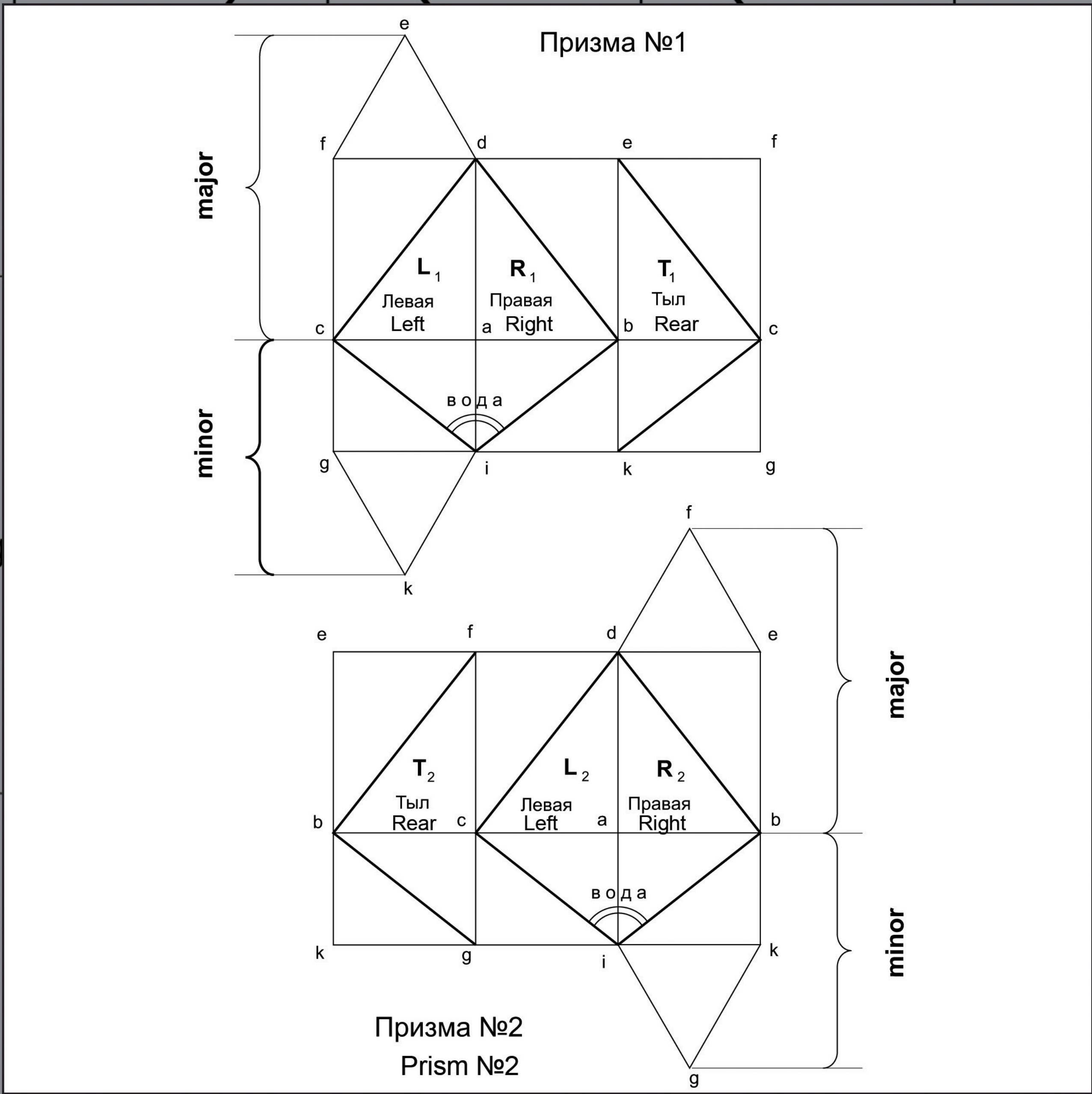
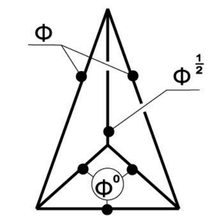
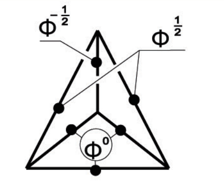
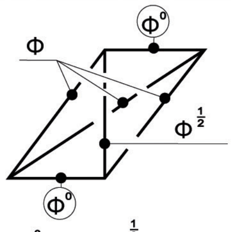
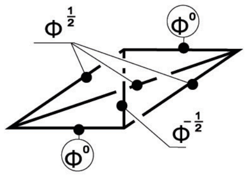
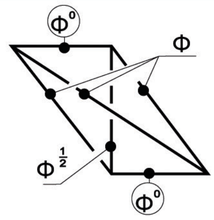
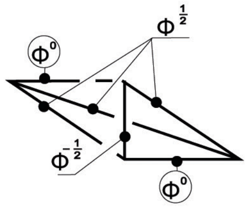


Рисунок
Figure 12.4

Слой **N** (major)		Слой **O** (minor)	
A $V_A = (\Phi^{\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1836002	 $\frac{\Phi^0}{1.000}$ 3 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 1 · $\frac{\Phi}{1.618}$ 2	B $V_B = (\Phi^{-\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1134712	 $\frac{\Phi^{-\frac{1}{2}}}{0.786}$ 1 · $\frac{\Phi^0}{1.000}$ 3 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 2
$D_{(+)}$ $V_{D_{(+)}} = (\Phi^{\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1836002	 $\frac{\Phi^0}{1.000}$ 2 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 1 · $\frac{\Phi}{1.618}$ 3	$C_{(+)}$ $V_{C_{(+)}} = (\Phi^{-\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1134712	 $\frac{\Phi^{-\frac{1}{2}}}{0.786}$ 1 · $\frac{\Phi^0}{1.000}$ 2 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 3
$D_{(-)}$ $V_{D_{(-)}} = (\Phi^{\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1836002	 $\frac{\Phi^0}{1.000}$ 2 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 1 · $\frac{\Phi}{1.618}$ 3	$C_{(-)}$ $V_{C_{(-)}} = (\Phi^{-\frac{1}{2}} \times 3^{\frac{1}{2}}) \times \frac{1}{12}$ 0.1134712	 $\frac{\Phi^{-\frac{1}{2}}}{0.786}$ 1 · $\frac{\Phi^0}{1.000}$ 2 · $\frac{\Phi^{\frac{1}{2}}}{1.272}$ 3

$V_A : V_{B_{(+)}} = \Phi$ $\qquad$ $V_A : V_{B_{(-)}} = \Phi$

Рис. 8 Каталог тетраэдров.
Fig. 8 Catalogue of Tetrahedrons

Рисунок Figure 12.5

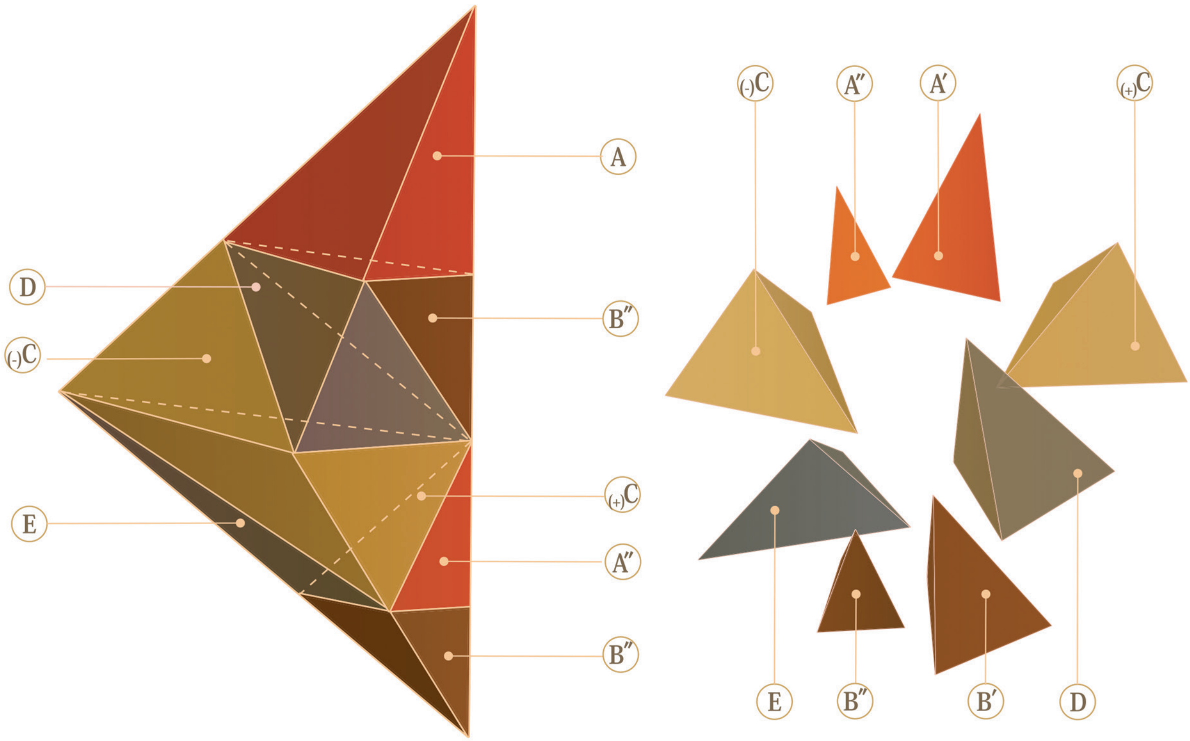


Рис. 28. ***Модуль пространства симметрии подобий, А-ромб***

a/ Тетраэдр **А**. Погружение в глубину в ритме $\sqrt{\Phi}$.
b/ Элементы А-ромба: тетраэдры А, В, С, D, Е.

Fig. 28. ***A module of the symmetry-of-similarities space, A-type rhomb***

a/ Tetrahedron A. Downsinking in the rhythm of $\sqrt{\Phi}$.
b/ A-rhomb elements: tetrahedrons A, B, C, D, E

Рисунок
Figure 13.1

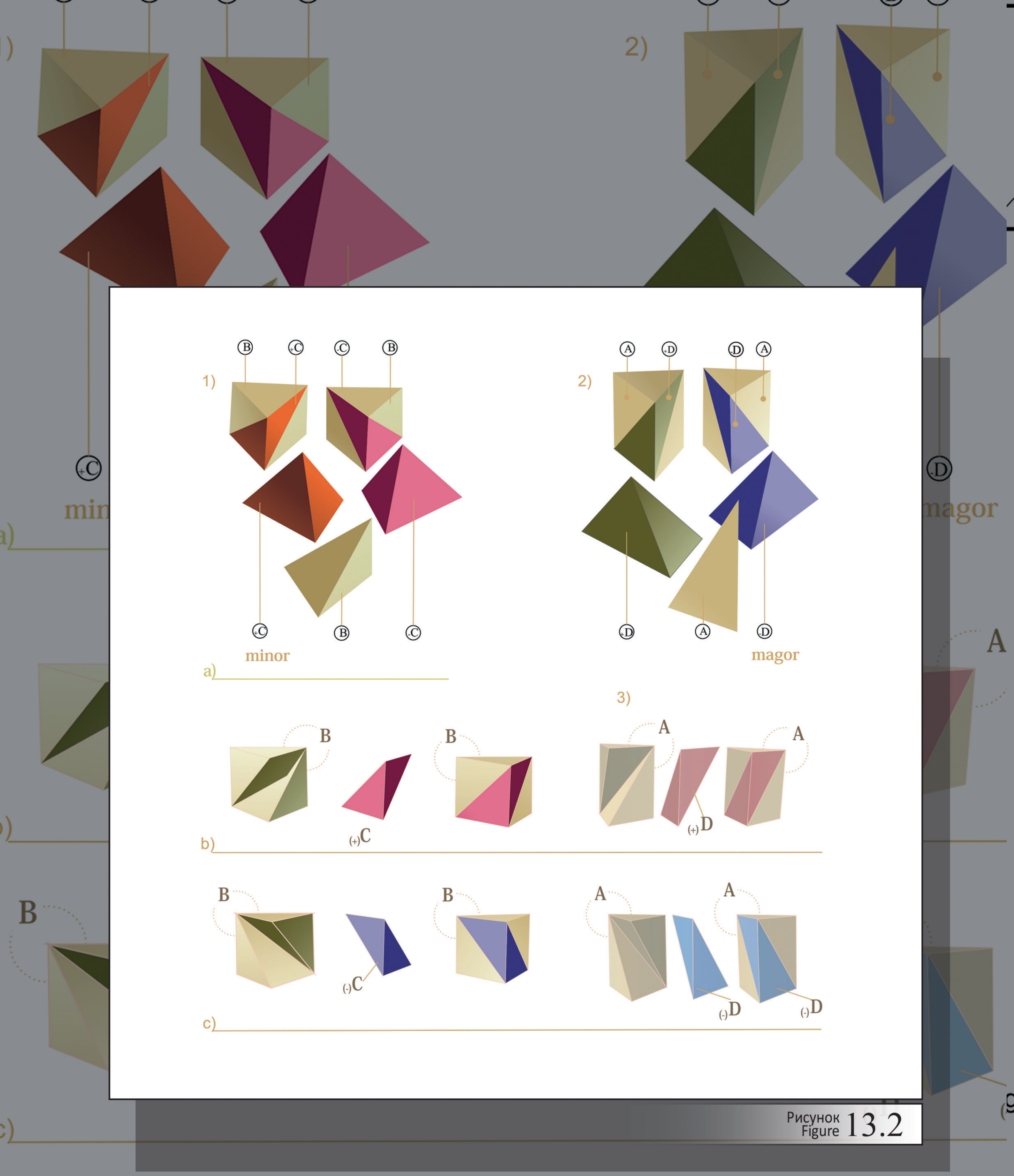


Рис. 24. ***Трехгранная призма и составляющие ее тетраэдры*** **B, $_{(+)}$C, $_{(-)}$C** (1, ***minor***).
и **A, $_{(+)}$D, $_{(-)}$D,** (2, ***major***)

a/ Общий вид и составляющие элементы.

b/ два тетраэдра B и один $_{(+)}$C, или два тетраэдра A и один $_{(+)}$D – правовращающая структу

c/ два тетраэдра B и один $_{(-)}$C, или два тетраэдры A и один $_{(-)}$D – левовращающая структу

Fig. 24. ***A trihedron prism and its constituent tetrahedrons*** B, $_{(+)}$C, $_{(-)}$C (1, ***the minor one***) ***and***
A, $_{(+)}$D, $_{(-)}$D, (2, ***the major one***)

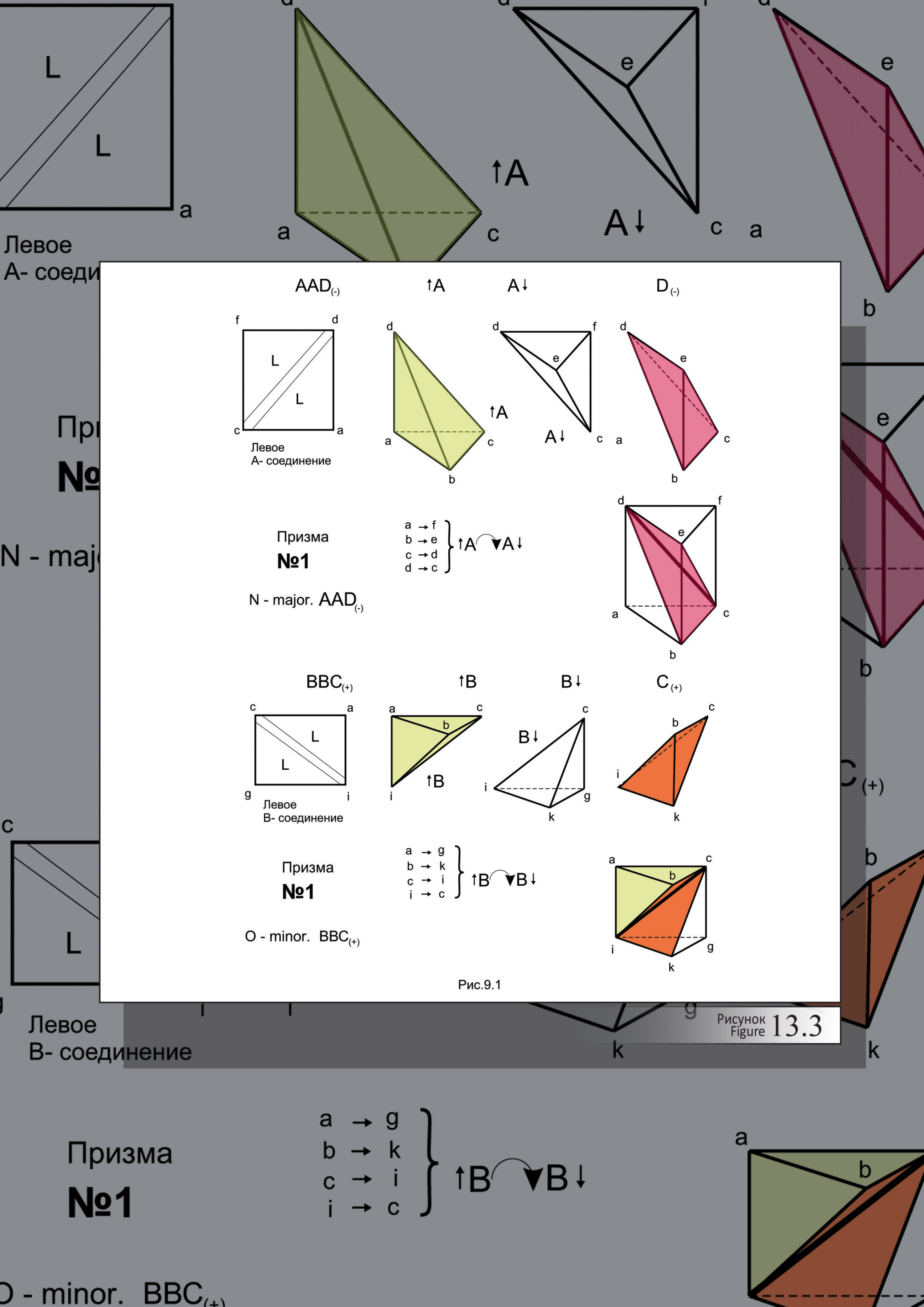


Рис.9.1

Рисунок Figure 13.3

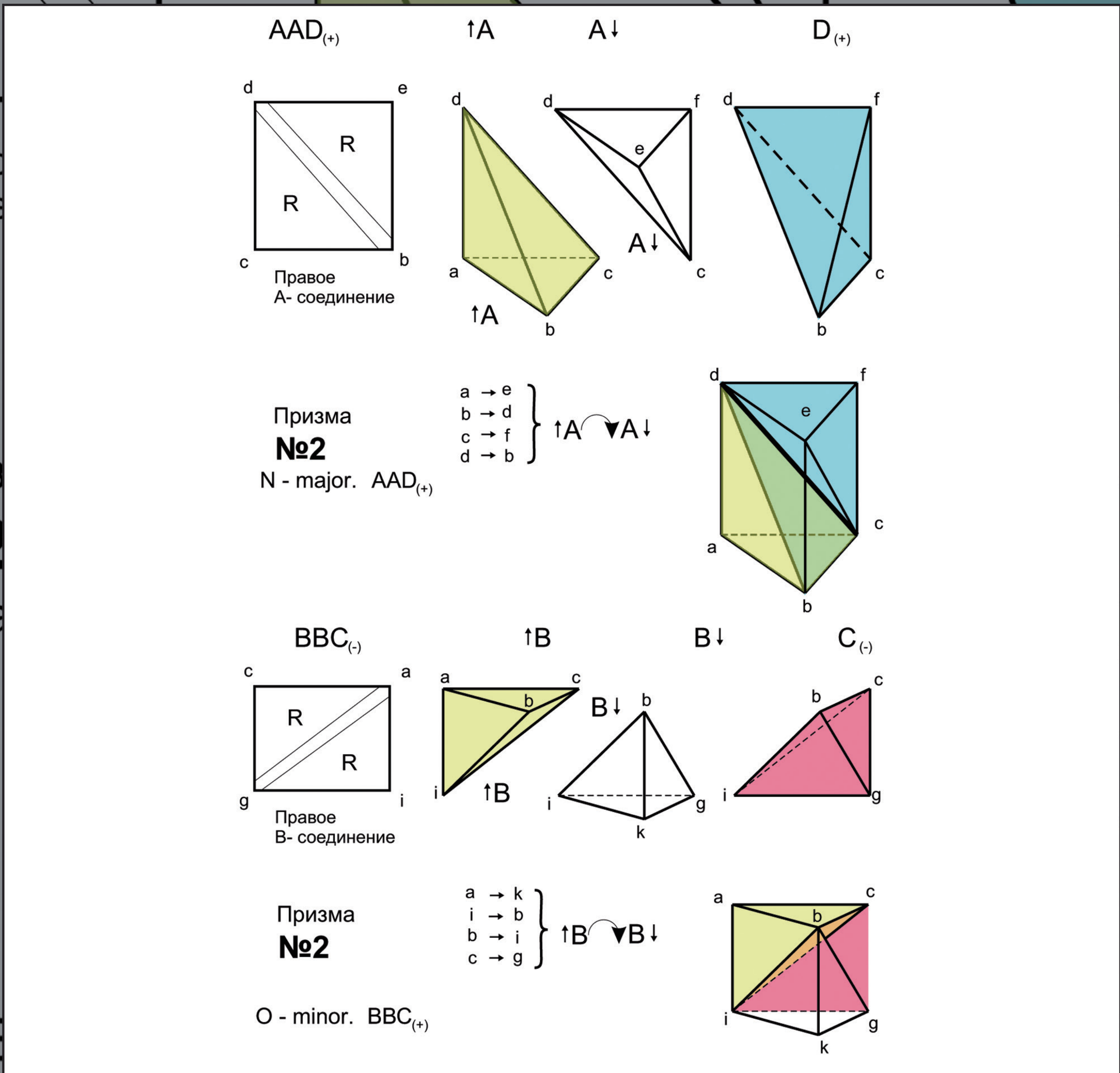


Рис.9.2

Рисунок
Figure 13.4

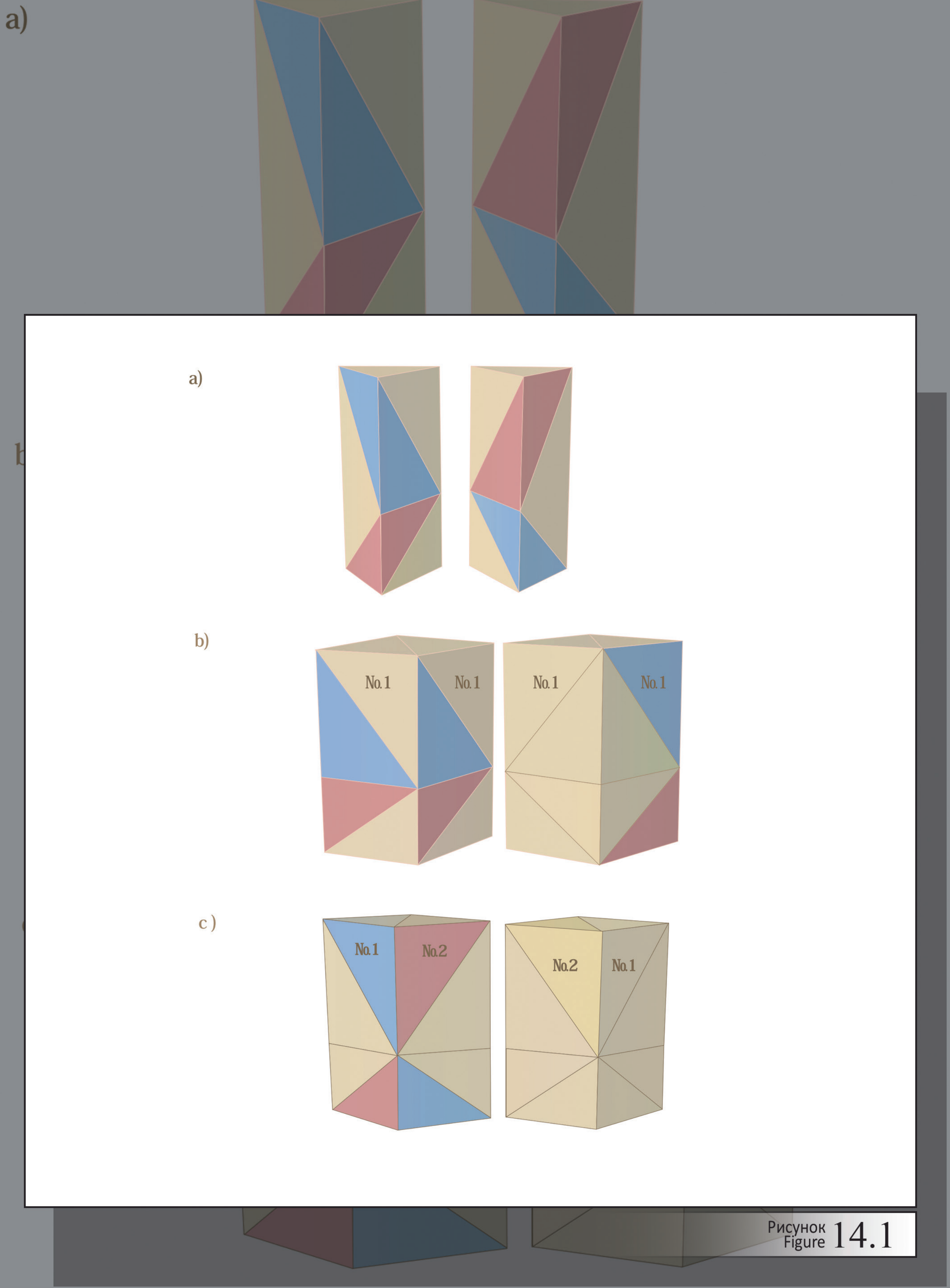


Рис. 29. ***Блокировка по вертикали***
a/ двухслойная призма minor-major . Структуры №1 и №2 зеркальны.
b/ двухслойная спаренная призма № 1 - № 1.
c/ двухслойная спаренная призма № 1 - № 2.

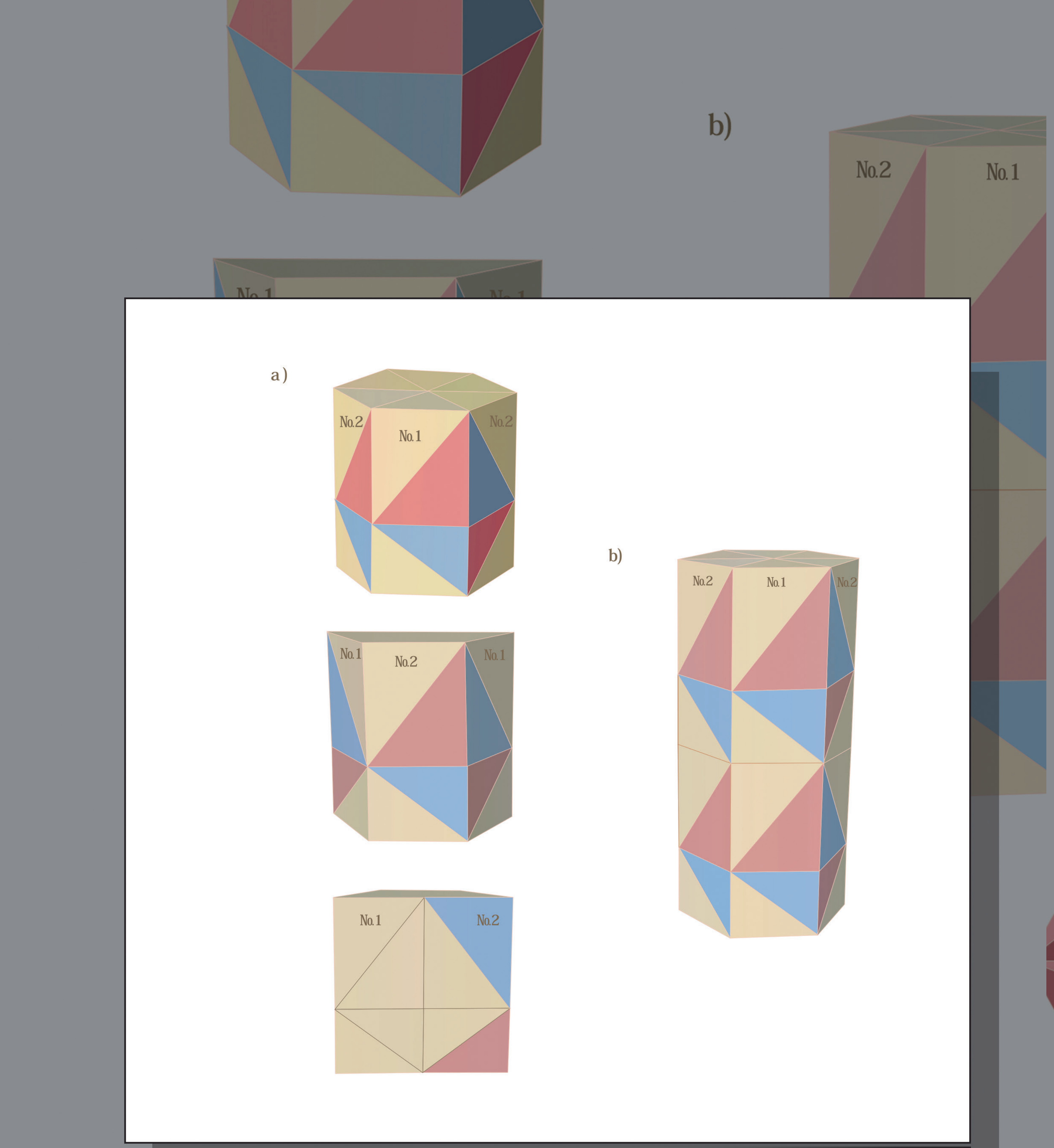


Рисунок Figure 14.2

Рис.30. ***Блокировка по вертикали***

Шестигранная призма minor-major составлена из трехгранных призм № 1 и № 2

a/ Двухслойная структура.

b/ Четырехслойная структура.

Fig. 30. ***Vertical interconnection***

A hexahedral minor/major type prism composed of trihedrons 1 and 2

a/ Two-layer structure

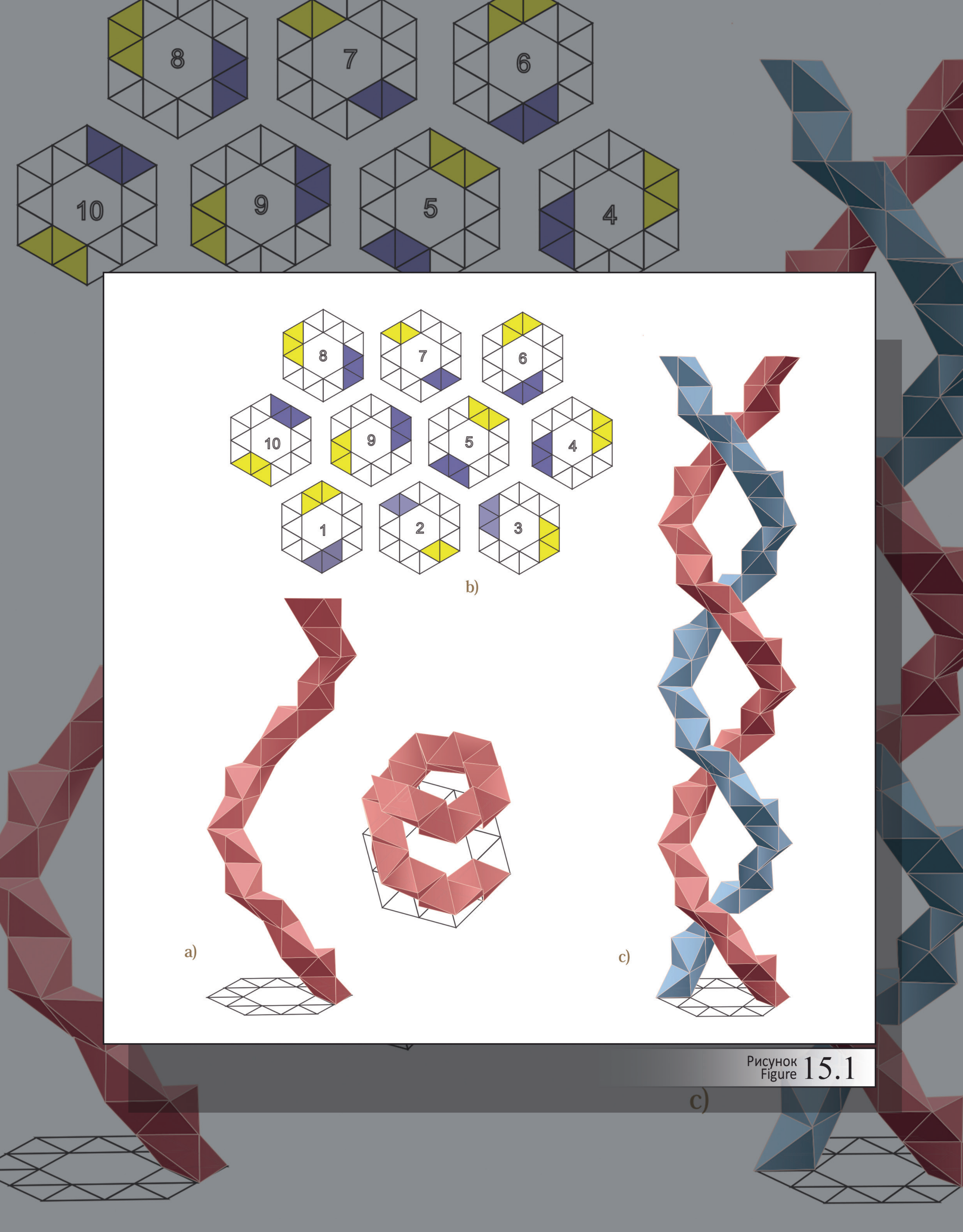


Рисунок Figure 15.1

6. ***Minor. Спираль двойная правильная правая, десяти витковая***

ктурный аналог В-спирали Крика-Уотсона (молекула ДНК).

бщий вид спирали... b/ План расположения по слоям элементов блокировки; это «

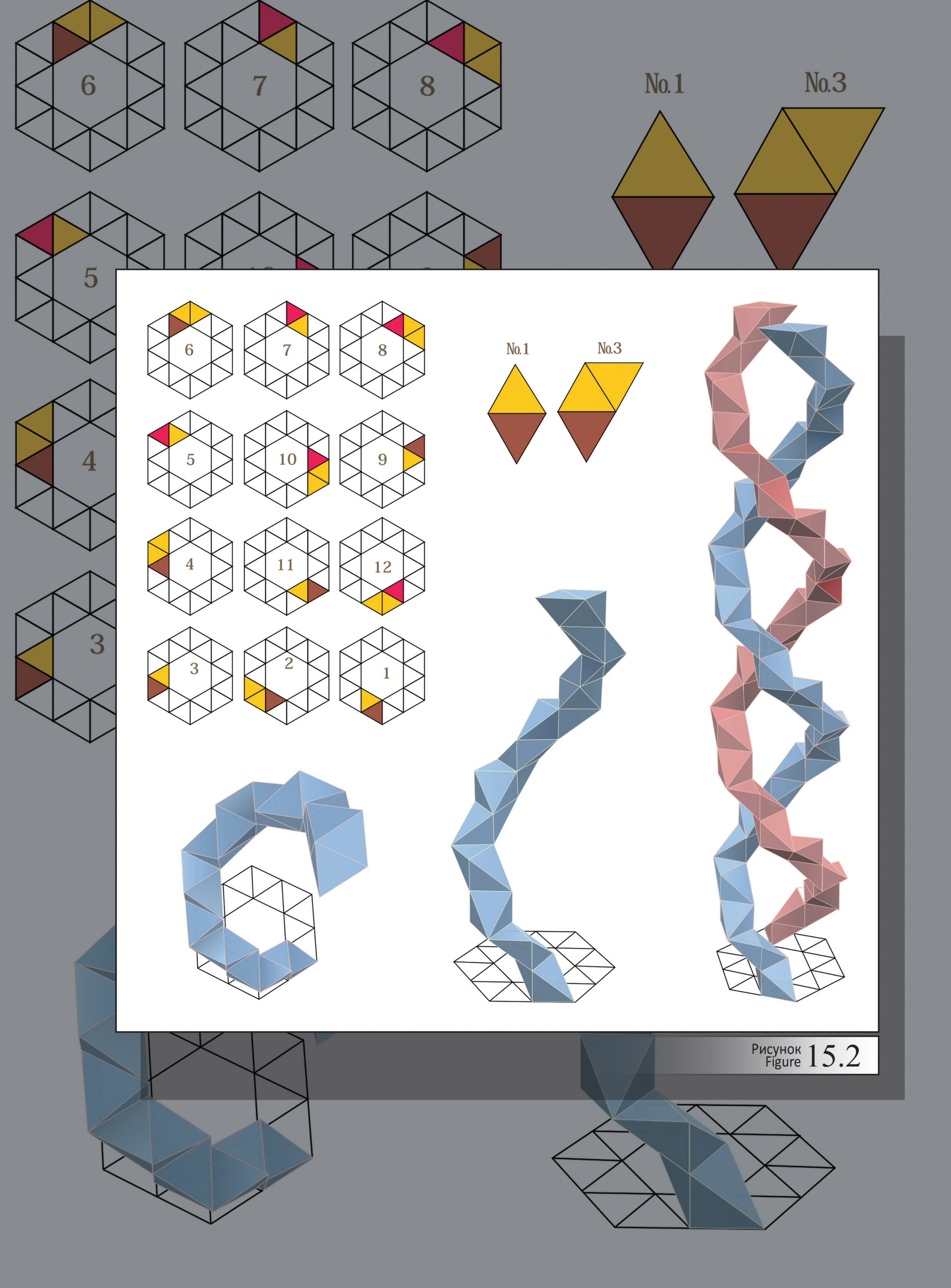


Рис. 35. ***Minor. Спираль двойная правильная левая, двенадцати витковая***

MASTER’S TOOL

Natural sciences broke apart crystals, live cells and subparts of the units of live and lifeless substance. On this way there occurred a great breakthrough in electronics and biology. But the general principle of the units-of-being form-building – the secret of harmony – is inaccessible to natural sciences. It is the field of art and mathematical sciences. Geometry is an area of insight into the laws of integrity in the great and small. Geometry speaks in language of numbers. *The number implies commensurating*. The notion of number always implicates a pair of numbers.

To compare the size of a piece of wood or stone to a palm span; to commensurate a leap of a dangerous beast and run of his own feet; to grasp the phenomenon of geometrical similarity, so as to draw, create symbols, – those are the tools of brain. The language of signs is a corner stone of civilization. The hand, step and foot became standard units of commensuration. Commensurating supplies us with information on world around, – and consequently, "*the World is number".* The Unit – a symmetry-of-pairs code ≡ the Second Pythagorean Theorem – is not a phantom of speculative thinkers, contributors to the problem of harmony. It is the very history of civilization. The epoch of blossoming sacral knowledge has left convincing, incontestable evidences for that.

The compasses from the Therm Museum in Rome – master’s workplace tool – and the proportional tree of Parthenon

Proportional compasses, whatever their aperture may be, feature two reciprocal numbers: $\frac{\mathbf{1}}{\boldsymbol{\alpha}} \Leftrightarrow \frac{\boldsymbol{\alpha}}{\mathbf{1}}$, which embody *αναλογια*, i.e. proportion. Two pivotally crosswise connected legs of such compasses expand and create two similar isosceles triangles. The distances between pointed ends of two paired legs on the extremities of compasses are the third, invisible bases of two similar triangles. The compasses represent an immense technology which deserve a good look through. Essential prerequisites for such technology are high culture and workmanship in executing due operations at a proper site, the feeling, insight and understood sequence of operations. Understanding the procedures involving the use of the Therm Museum compasses at creation of the Parthenon dimensional-space structure casts some doubt on current views regarding a role of proportions in creative process and contravenes the principles (if any!) of applying proportions by the modern architectural school. It is my duty to leave for the future a lesson given by Phidias and to show how the Master works.

We know four antique proportional compasses.[15] Two of them are set on doubling, 1/2 = 0.500. The third ones, glorified, stored in the Neapolitan Museum of Arts, are set to the ratio of the Golden Section, 1/Φ = 0.618. **The fourth** – those from the Therm Museum in Rome – reproduce ratio ($\sqrt{\mathbf{5}}$ - 1) / $\sqrt{\mathbf{5}}$ = 0.553.

The Neapolitan "golden" compasses have been discovered in a workshop *of the sculptor* in Pompey, and many see it as a convenient tool for harmonization of the form. But it is not the whole truth. Art is inseparable from figurative associations. The Golden Section is an impersonal principle, unrelated to such associations. It is universal. The Greeks supposed the Gods to be similar to people in all respects, but much more powerful. The key to universal gamma of ratios capable, with the use of simple techniques, to reproduce in stone the tenfold scaled-up image of a man, give compasses from the Therm Museum in Rome (Fig. 16.1). They

[15] Брунов Н.И. Пропорции античной и средневековой архитектуры. Москва, 1935 (Brunov N. I.; *Proportions of the Antique and Medieval Architecture*; Moscow, 1935).

employ, as one of linking alternatives, the Golden Section. Possibly, the Therm compasses are an exact copy of those used by the Parthenon builders.

Application rules of any proportional compasses are elementary. There are only two techniques:

1) *Commensuration of sizes.* At downscaling, the initial size is set by a prick of long legs; the sought-for size is found by a prick of short legs. At upscaling, the initial size is set by a prick of short legs; the sought-for size is found by a prick of long legs.

2) *Doubling of sizes* is achieved by half-round turning the compasses.

What is the effect of these simple techniques? *One commensuration* plus *one doubling* realized by proportional compasses from the Therm Museum in Rome build a proportional scale necessary and sufficient to achieve dynamic equilibrium of the masterpiece dimensional structure in all its details. The proportional scale created by this procedure is presented on Fig. 16.1. You see there segment ***bc' =*** 1.447. Points ***a, c*** divide this segment into three parts):

bc = 0.553 , ***ac =*** 0.447, ***ac'*** = 0.447

The golden octave of interpenetrating similarities of the double square system[16], which is necessary and sufficient to the architect, is reproduced exhaustingly:

1) identity	***ca: ac'*** =	1.000 =	1/1
2) doubling/dichotomy	***ca: cc'*** =	0.500 =	1/2
3) Golden Section, or "*the first invariable*"	***cc': c'b*** = (0.447 x 2):1/447 =	0.618 =	1/Φ
4) Golden Section squared	***bc:bc'*** =	0.382	$1/\Phi^2$
5) half-golden	***ac':bc*** =	0.309	$\Phi^{-1}/2$
6) double golden	***ac:cb*** =	0.809	Φ/2
7) quinary symmetry	***ca:ab*** =	0.447	$1/\sqrt{5}$
8) "*the second invariable*"	***cc':ab*** =	0.894	$2/\sqrt{5}$

The golden octave is resulted from juxtaposing *three numbers* (**Φ,** $\sqrt{5}$ **, 2**) to number **1**. So in optics and painting: three conjoining colors give *"light", i.e.* the white color (1), but when intermingling, they make all remaining colors palette. Analogy is perfect. More than that, eight mentioned pairs of numbers are similar to eight sound steps of the musical octave.

To possess a tool is not the same as to command it. The proportion is correspondence of an art object as a whole, and correspondence of all its members as well, to a part accepted as a basic, initial unit. The proportion is "*αναλογια*". The Parthenon is likened to a tenfold scaled-up man. The growth of "a well knit man" is *six feet*. Human foot makes 1 *ft,* its length is equal to that of a head with neck, *five foots* make the height *of human body* measured from the bottom of planta to sternal notch in the base of neck. The Greeks named the column shaft (a symbol of *slenderness, beauty and load-bearing ability*) with a word *"σομα"* that literally means "*body*". The column capital (a *head*) in the Doric order, as a part of structure, is a pad at the junction of architrave stones. This being the case, and as stated by Socrates, the son of a stonemason, descending from Daedalus generation and being stonemason and sculptor in his own youth, –

[16] И. Шевелев. Золотое пространство. Кострома. Промдизайн-М, 2006. с. 26-27 и 42-49. (J. Shevelev; *The Golden Space*; Promdesign-M Publishers. Kostroma, 2006, p. 26-27 and 42-49).

"*the best connection afford mean ratios*" , – proportion of the Parthenon was defined by number $\sqrt{5}$, the mean proportional of numbers 1 and 5:

$$1:\sqrt{5} = \sqrt{5} : 5 = 0.447$$

It is extremely important to note: taking human body as a model of strength and beauty (1/5), the master not only subordinated to this ratio proportionality of the column shaft, but also *extended this link to proportionality of the column as a whole, including the capital*. The mean proportional of numbers 1 and 5, link $1:\sqrt{5}$, *was applied by analogy* to many other parts of the temple, ranging from the link between a 100-ft stylobate width and its length to such small details as the height of a cornice plate, height of hypotrachelium, depth of flutes. Those links were applied not in the same key, but ingeniously, in a variety of ways, as appropriate to the nature. The Greeks understood the force of polyphony. The main theme, *quinary* proportion ***1/*** $\sqrt{5}$ ***= 0.447***, was applied 6 times from 11 where it might be necessary; the second one, *golden* proportion, – four times, and in a rather original manner. Where a contrasting link between the column shaft height and stylobate width was necessary, and where it was desired to reinforce the colonnade power (at temple corners), the master had twice applied *semigolden* proportion 1/2Φ = ***0.309***: firstly, as a ratio of the column shaft height to stylobate width, and for the second time – as a ratio of the truncated corner column spacing to column shaft height. Moreover, he connected the total load on column shafts (considering total height of the capital, entablature and fronton) with the column shaft height by means of link Φ/2 = 0.809 (*double golden proportion*) (Fig. 16.1, 3, 4). The third theme: *the second invariable*, ***2/***$\sqrt{5}$ ***= 0.894***, connected the truncated corner column spacing to the height of entablature (Fig. 16.4).

Collapse of "overthrowing" the double square system

"The connection of parts and whole established by you in the Parthenon on the Athenian Acropolis is convincing, exact, is more than beautiful!" – such words could tell me diligent professionals/opponents. But they kept mum. As if there were no publications.[17] Instead they have noticed inaccuracies. Where I see number $1/\sqrt{5}$ = 0.447 (stylobate commensurability, the ratio of the column diameter to column spacing, subdividing entablement into architrave, frieze and cornice, etc.), exact measurements discover integer-valued ratios. Sometimes 4:9 = 0.444, sometimes 31:69 = 0.449. Therefore the opponents believe in integers and, sometimes, in the Golden Section. But they refuse to believe logic of classical antiquity, which had explicitly, univalently, once and for all identified concepts of *analogy* (likeness) and *proportion* (number). The academic science (the theory and history of architecture) has denied intelligence to great Phidias and all his followers (as well as Egyptology – to builders of pyramids), crediting them not with bright reason, but with senseless manipulations with integer-valued ratios of unknown origin.

To establish the truth, I ask two questions: 1. Why is the height of the Parthenon column shaft equal to 31 ft (as noticed by Andrey Chernov), instead of e.g. 30 or 36 ft? 2. How could the master embody in stone the great *geometrical* conception, using labor of many tens masons and other builders, without a generally accepted standard of measure? And I myself furnish an answer.

[17] Lib.: Геометрическая гармония, 1963. (*Geometrical Harmony*, 1963). Magazines: "Наука и жизнь", 1965, № 8; "Архитектура СССР", 1965, № 3, and many more.

1. The Parthenon is a 100-ft temple (the "100 ft" is stylobate width). The occurrence of a 31-ft dimension in the 100-ft temple is due to the following reasons: firstly, 31 + 69 = 100, and secondly, 31:69 = $1/\sqrt{5}$. Integer-valued relationship 31:69 = 0.449 offers a rather close approximation of number 0.447. Considering that any architectural form is geometrical, it becomes obvious: we are facing the geometry. The ratio of a double square side to its diagonal is the main factor. This is evidenced by the right-angled ground plan of stylobate, where colonnade plate axes, locations of each column and each wall of the temple have been defined and drawn.

2. The answer to the second question is not so simple. Two circumstances are to be noted. To begin with, architecture is not bookkeeping. The impression of harmony is due to singularities of perception. Breath is peculiar to architectural forms. A column is diminished, i.e. its diameter decreases upwards (entasis), and the generatrix of shaft is made bent so as to appear as a straight line. Stylobate is curved, it raises to the curve center; the corner columns are thicker than ordinary columns, etc. *The Parthenon proportion is by necessity divaricated.* The number fluctuates as a sounding string. Therefore the form lives. The width of abacus of an ordinary column varies within several centimeters. The difference in thickness between ordinary and corner columns is 42 mm.[18]

Secondly, building process is inconceivable without application of a measure. The measure is a language interconnecting people and material. Number $1/\sqrt{5}$ = 0.447 has two magnificent integer-valued approximations. The first of them sets contrast high (– 0.003), it is "$\sqrt{5}$-*diesis*", 4:9 = 0.444; the second reduces contrast (+ 0.002), "$\sqrt{5}$-*bemolle*", 31:69 = 0.449.

And we see: moving away **from** geometrical idea of interpenetrating commensurabilities linked by a chain of analogies **to** *integers* was favorable for builders and did not put sudden obstructions for the master, who ingeniously used this divarication as a means for spiritualizing and anthropomorphization of stone.

The Parthenon is, firstly, an *idea*, image; secondly, it is *material* (pentelic marble of a warm flesh-color associated with human body); thirdly, it is embodiment – *a method of* $\alpha\nu\alpha\lambda o\gamma\iota\alpha$.

The man is a perfect unit of the nature. From here also made its appearance the Parthenon, a hymn the quinary symmetry, to life. A hymn to the human body inheriting the Golden Section:

$$\sqrt{5}/1 = (\frac{\Phi}{1} + \frac{1}{\Phi})/1.$$

Hence, "in the beginning was the ***λογοσ***": the Word-number. The form of Parthenon spontaneously showed an image of metaphysical Unit $\frac{\Phi}{1} + \frac{1}{\Phi}$ to which my book is devoted. It embodied the Unit in marble of the perfect Doric temple colonnade. The temple is created after the image and likeness of man. The idea dominates in the act of creation. So, "in the beginning was the ***λογοσ***": the Word-number.

The World is number! And without him was not any thing made that was made.

The proportional compasses from the Therm Museum in Rome build *eight numerical ratios* – a complete "octave of accords", which is sufficient for definition of dimensional structure of the Athenian Acropolis temples, by a sole measurement of initial value and two

[18] The Therm Museum compasses make clear this difference: the mean design value of diameter $\frac{1.901+1.943}{2}$ = 1.922 m was modified by two corrections: 13 mm and 29 mm. 13:29 = $1/\sqrt{5}$. An ordinary column is more slender, its diameter equals 1.914 m – 0.013 m = 1.901 m; an angular column is thicker, its diameter equals 1.914 m + 0,029 m = 1.943 m.

pricks of inverse legs. Connection leitmotif $1/\sqrt{5} = 0.447$ integrated sizes of parts and whole in the manful Parthenon, whereas a proportion leitmotif of the womanlike Erechtheum[19] is the ratio $2/\sqrt{5} = 0.894$.

A tool capable to build rectangles composed of each other (i.e. to create a gamma of commensurabilities) and thereby create sensation of harmony, integrate parts in a whole, – is at all times in-demand, intentionally or semi-consciously. Antiquity had created compasses. The Middle Ages have generated measuring cane[20]. In the middle of 20th century a spall of such cane is found by A. Mongait's archaeological expedition in ancient Novgorod, in an occupation stratum of the early 12th century.

Basically, it is a conjugate measure: two proportional compasses joined together. One of four faces of the Novgorod measured cane is void, and on both its sides – on three remaining faces – are plotted biconjugated size scales. First pair of scales reproduces double golden proportion $\Phi/2 = 0.809$ including the measuring sazhen (full reach of arms) and Tmutarakan sazhen (the double step). Second pair of scales reproduces ratio $1/\sqrt{2} = 0.707$ with the Tmutarakan oblique and Novgorod oblique sazhens. The proportions of medieval Russia temples and those of the Ascension Church in Kolomna, outside of Moscow, are related to the described or a similar measuring cane.[21] Measuring cane is a tool meant not only for developing conceptions (just as compasses), but also for work on construction site. Thus the problem of incommensurability of a side and diagonal of square and double square is automatically solved. The question: how the master on a construction site will pass over from geometry to integer-valued ratios? – does not come up. The problem is solved *by the equal or doubled counting of units which are read off on two geometrically conjugated scales.*

It is interesting to grasp meaning in the philosophy of Units. Let us try to relate the origin of the Therm compasses to the truth that the nature of Unit is binary.

$$\text{Unit } \Phi = \tfrac{1}{2}\sqrt{5} + \tfrac{1}{2}\,1$$

$$\text{Unit } \sqrt{5} = \Phi/1 + 1/\Phi$$

$$\text{Unit } 1 = \Phi/1 - 1/\Phi$$

Dividing Unit 1 in two *equal* parts generates numbers $1 = \frac{1}{2} + \frac{1}{2}$. The ratio of the Therm Museum compasses, **0.553**, results from division of unit 1 in two *unequal* parts, in a dynamic ratio: **1** = **0.553 + 0.447**. It is logical to assume that "the second half" of unit 1, the proportional compasses 0.447 explicitly set on a theme of "man", also existed. Employing one commensuration and a prick of inverse legs either side: $(1 + \sqrt{5}^{-1})$ and $(1 - \sqrt{5}^{-1})$, the imaginary compasses 0.447 draw picture every bit as presented above: *the golden octave,* eight ratios necessary for the master (Fig. 16.1). It is remarkable that compasses 0.447 really existed, 3200 years prior to birth of Phidias! This is confirmed by an image of conjugate measure, a relief, skillfully cut out on a wood board amazingly preserved over a span of almost five millenaries[22]. We see there two measuring canes interconnected lengthwise as 1 and $\sqrt{5}$ in the arm of architect Hesi-Ra, the builder of the first Egyptian step-pyramid (Fig. 16.3).

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[19] Шевелев И. Ш. Принцип пропорции. М., Стройиздат. 1984. стр.96-106 (J. Shevelev; The Principle of Proportion; M., Stroyizdat. 1984, pp.96-106).

[20] Modern architecture is also looking for combinatorial standards. The key is in our research.

[21] The first half of the 16th century. Ibid. pp.165-171.

[22] The connection between the lengths of the measuring rods on the carved panel depicting architect Hesi-Ra $1/\sqrt{5}$) in close connection with the Parthenon proportion, golden ratios, and the double square was traced by me and repeatedly published in 1962-1963.

All four widely known proportional compasses, and also the fifth, imagined but quite real ones (the Hesi-Ra measuring staffs) originate from a simple geometrical figure of double square. And precious numerical ratios concealed in the double square, – the ratios representing the Φ-structure as whole, – were materialized ages ago and kept for the future in a long-life material, granite. I do mean the inner space of the Pharaoh Cheops funeral chamber, a heart of the most grandiose and mysterious pyramid. The floor in this chamber is configured as a double square; the end wall reproduces the second constant of natural geometry, commensurability 2/ $\sqrt{5}$. Different combinations gained by addition of sizes inherent in faces of the funeral chamber have defined slope angles of facing of all ten great pyramids in the sacred complex of Giza. Just *in their diagonal sections*, instead of apothem sections. Because the edges converging to apex create the skyline of a pyramid, its image read against the sky. In practice, erection of facing starts with laying of the corner block (rib).[23] Let us denote pyramid height by **H** (measuring from the level of platform which supports the facing, up to the pyramid apex) and by **B** – projection of edge on the platform plane, following which we determine the mentioned diagonal sections (Fig. 16.6):

1)	H:B = 1:1		rhomboid Snofru pyramid (south), base portion
	H:B = 2	: (1+2)	rhomboid Snofru pyramid (south), top portion
2,3,4)	H:B = 2	: ($\sqrt{5}$ - 1)	Khuni, Cheops, Neuser-re pyramid
5,6,7)	H:B = (2 + 2)	: ($\sqrt{5}$ + 2)	Khafer, Nefer-ir-Kare and Pepi II pyramids
8)	H:B = (1 + 2)	: (2+2)	Mykerinos pyramid
9)	H:B = ($\sqrt{2}$ + 2)	: (2+2)	Sahure pyramid
10)	H:B = ($\sqrt{5}$ + 2)	: (2+2)	Unas pyramid

Inexhaustible energy of creativity hidden from time immemorial in the Double square boggles the mind. There is a natural problem. Why in a thousand-year history of pyramid building, the First triangular pyramid was the wisest one, and why, being prior in tempore, it hides inexhaustible sense in its core?

Why this sense is so affined to mathematical texture of constants and magnitudes of Natural geometry? To an image of three spheres nested into each other, to the kernel of the Φ-sphere Unit, which takes shape in the present-day ideas about structure of the world and in abstract speculations on the nature of number? We are facing, probably, with memory of a Great civilization. However interesting it may be, to know, what kind of Civilization it is, whether Terrestrial Atlantis, or alien, cosmic, – the overall objective is to penetrate as deep as possible into its ideas and knowledge using all ways accessible to our intelligence.

Kostroma
October 14, 2014

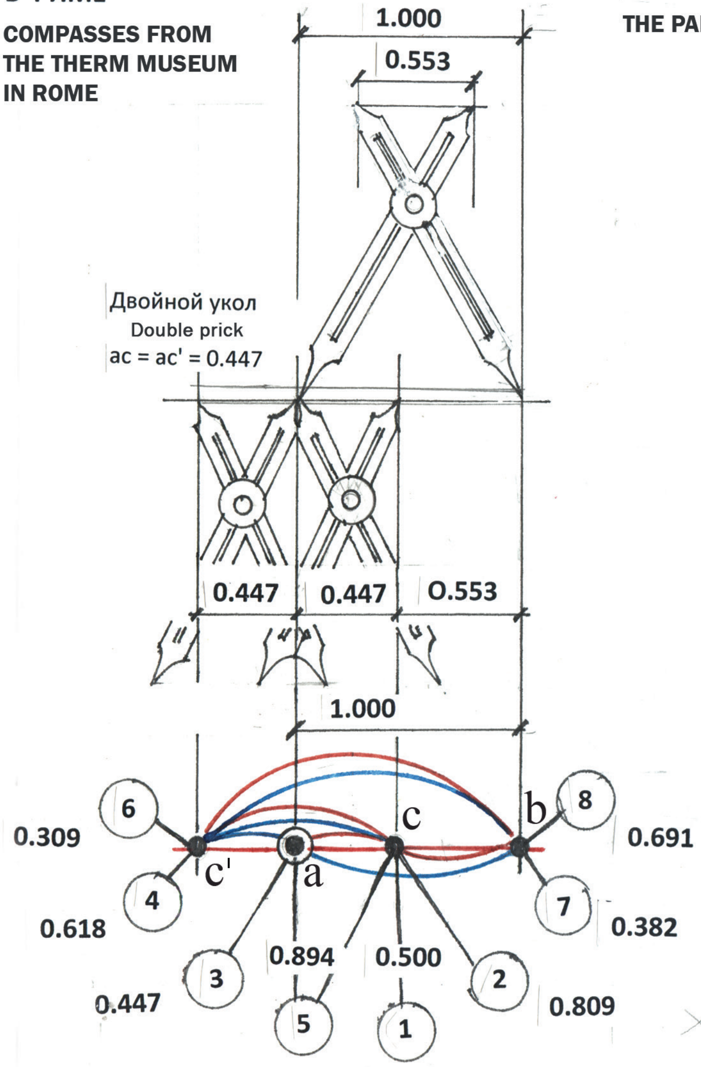
[23] For more details, see И. Шевелев. Основы гармонии. М. 2009. (J. Shevelev; The Fundamentals of Harmony; Chapters: At the Dawn of Civilization, Ancient Proportion, and Paired Measures in Ancient Rus; M. 2009;).

ИЛЛЮСТРАЦИИ К III ЧАСТИ

ILLUSTRATIONS PART III

ЦИРКУЛЬ МУЗЕЯ ТЕРМ В РИМЕ

COMPASSES FROM THE THERM MUSEUM IN ROME



ПАРФЕНОН АФИНСКОГО АКРОПОЛЯ

THE PARTHENON ON THE ATHENIAN ACROPOLIS

1. ширина стилобата Stylobate width	30.870м	
длина стилобата Stylobate length	69.516м	№ 3
2. ширина стилобата Stylobate width	30.870м	
высота ствола колонны Stylobate heigth	9.57	№ 6
3. высота ствола колонны Column shaft height	9.570м	
диаметр колонны (средний) Column diameter (ave.)	1.922м	
4. высота ствола колонны Column shaft height	9.570м	
шаг рядовых колонн Ordinary column spasing	4.295м	№ 3
5. высота ствола колонны Column shaft height	9.570м	
шаг угловой колонны (сев.) Corner column spasing (North)	3.662м	№ 6
6. диаметр колонны Column diameter	1.922м	
высота капители Capital height	0.860м	№ 3
7. шаг угловой колонны (юж.) Corner column spacing (South)	3.698м	
высота антаблемента Entablement height	3.297м	№ 5
8. высота антаблемента Entablement height	3.297м	
карниз Cornice	0.600м	
фриз (архитрав) Frieze (architrave)	1.350м	№ 3
9. высота капители Entablement height	0.860м	
шейка Collar	0,156м	
эхин, абак Echinus, abacus	0,352м	№ 3
10. завершение (нагрузка на ствол) Entablatere (load on shaft)		
Капит.+антабл.+фронтон Capital + antablement + fronton	7.735м	
высота ствола . Shaft height	9.570м	№ 2
11 глубина храм (в чистоте) Temple clear depth		
Парфенос Parthenon	13.363м	
Афины Athen	29.657 м	№ 3

Рисунок
Figure 16.1

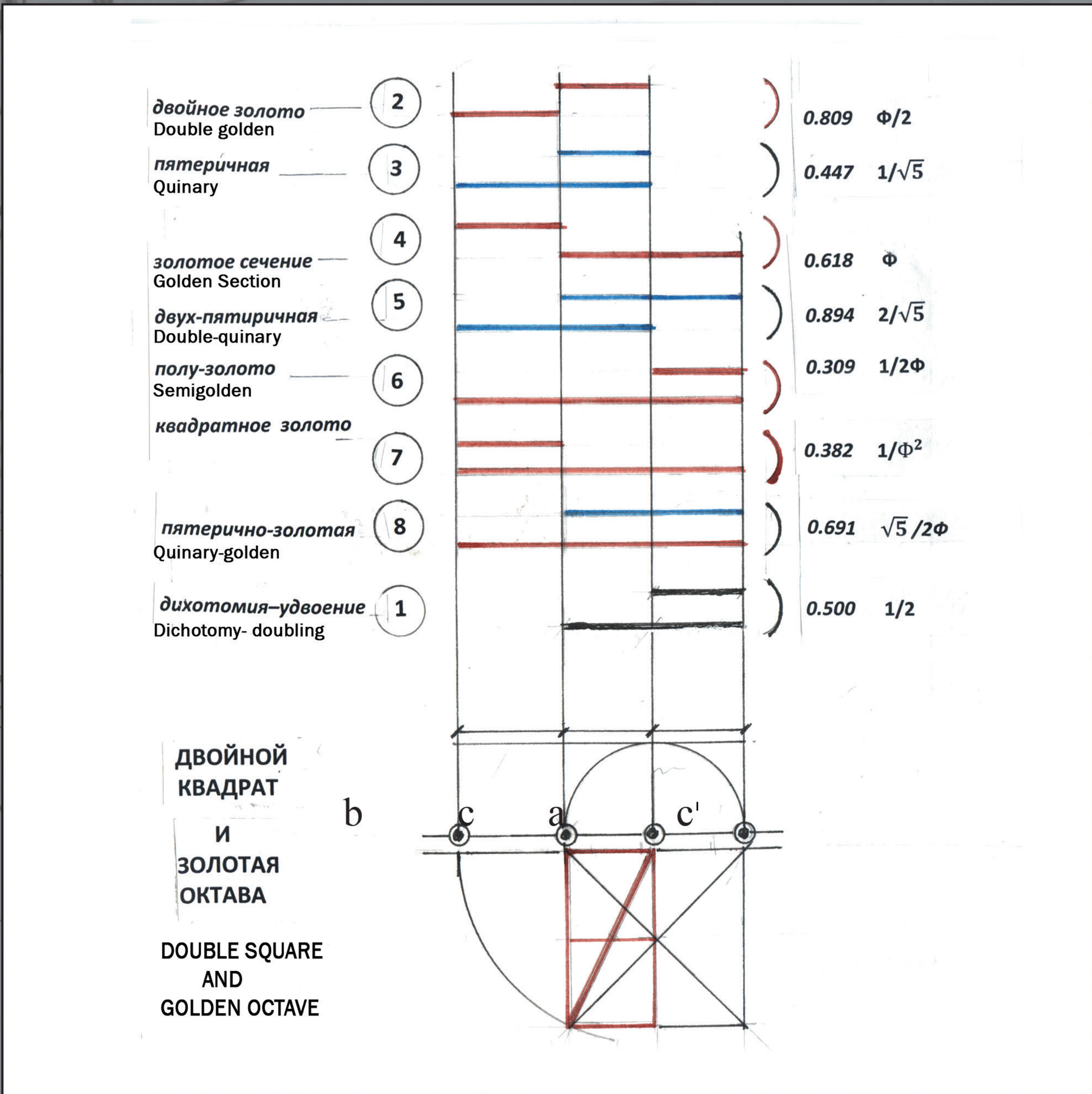


Рисунок
Figure 16.2

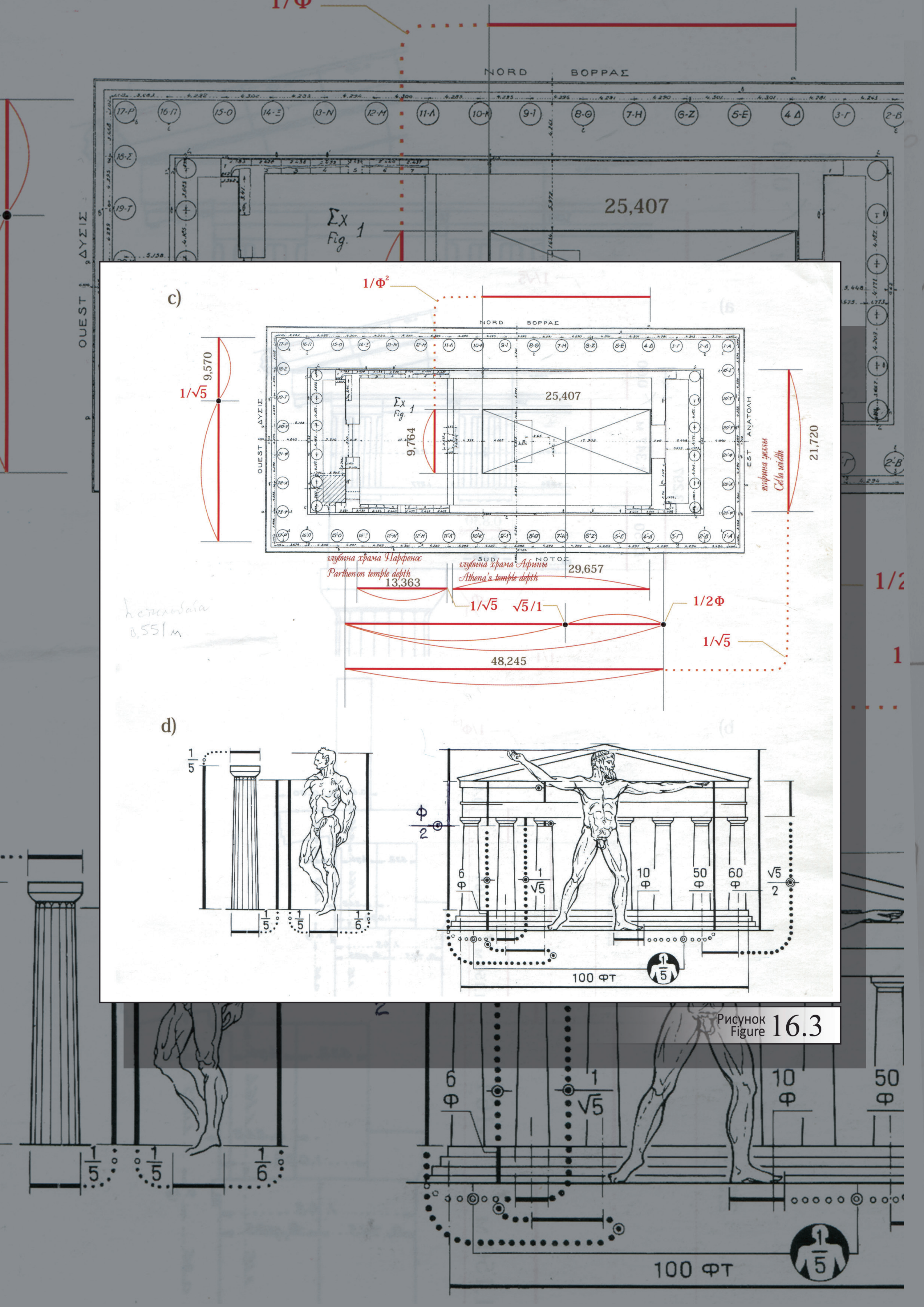


Рисунок
Figure 16.3

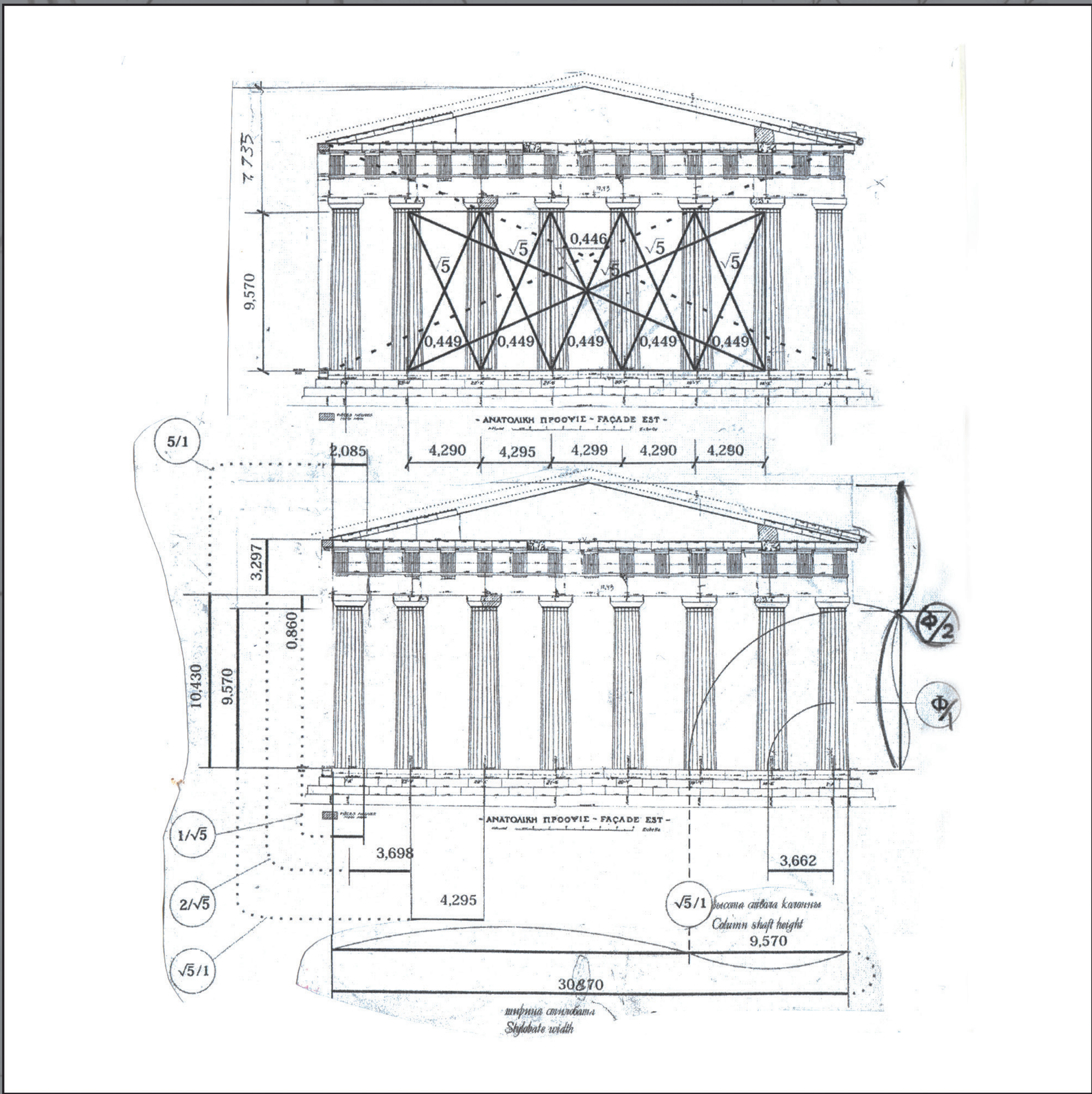
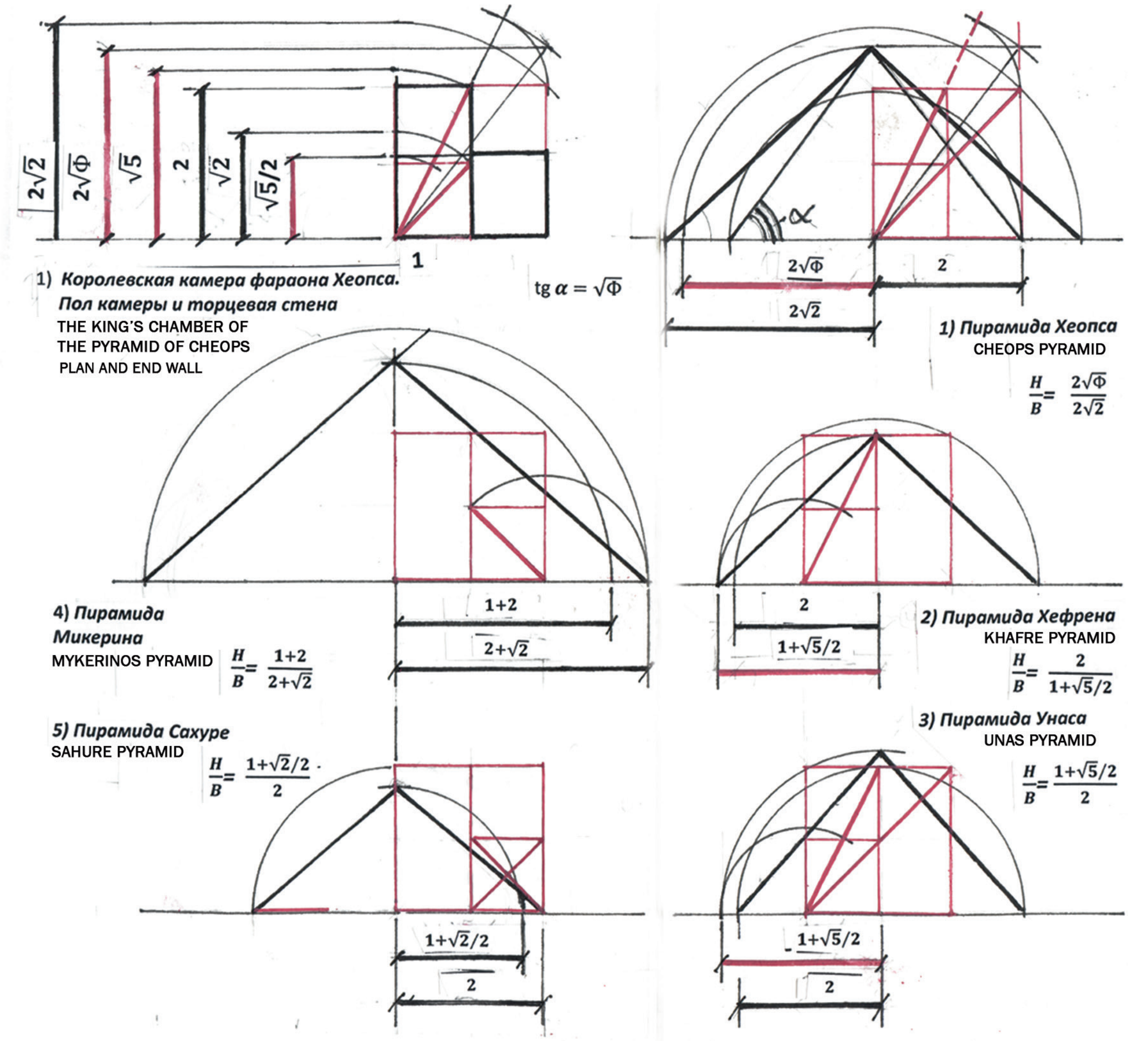


Рисунок
Figure 16.4

Рисунок Figure 16.5



ДИАГОНАЛЬНЫЕ СЕЧЕНИЯ ЕГТПЕТСКИХ ПИРАМИД ДРЕВНЕГО ЦАРСТВА

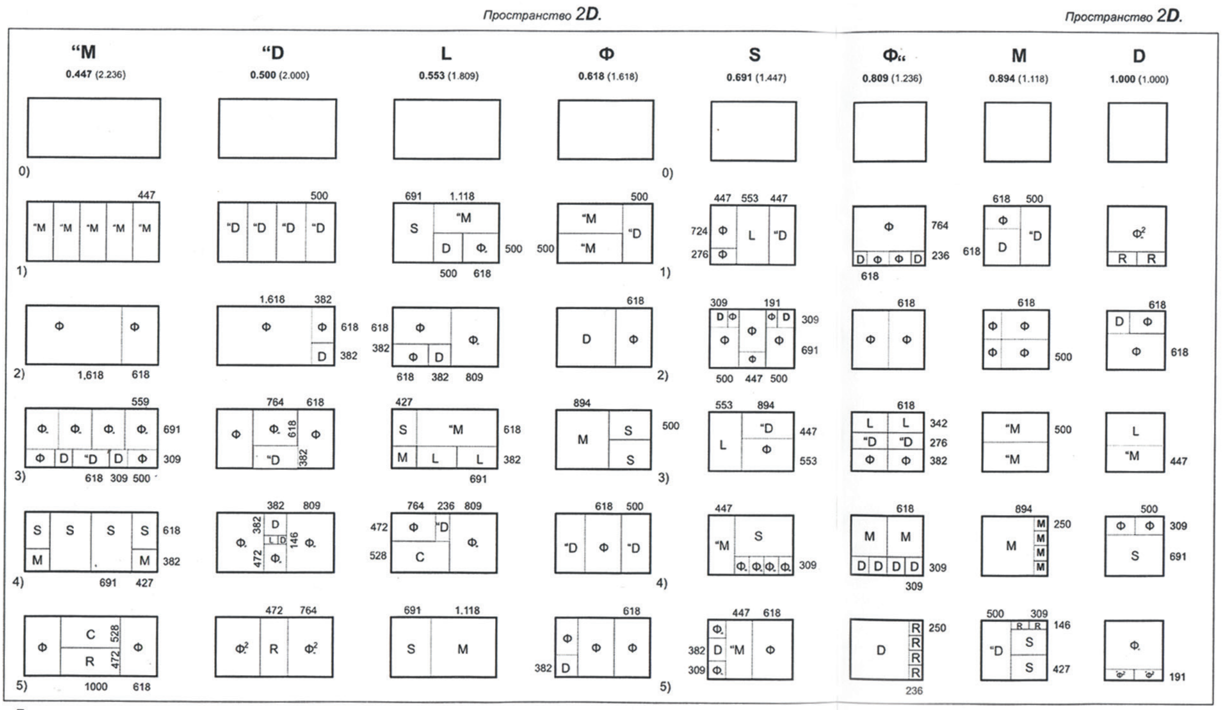
DIAGONAL SECTION DRAWINGS OF THE EGYPTIAN OLD KINGDOM PYRAMIDS

Рисунок Figure 16.6

Таблица 2. Взаимопроникающие подобия. Фрагменты.
Соразмерности 0,447, 0500, 0,553, 0,618.

Таблица 2. Взаимопроникающие подобия. Фрагменты.
Соразмерности 0,691 0,809, 0,894, 1,000.

Mutually interpenetrating similarities. Fragments. Commensurabilities 0.447, 0.500, 0.553, 0.618, 0.691, 0.809,0.894, 1.000.



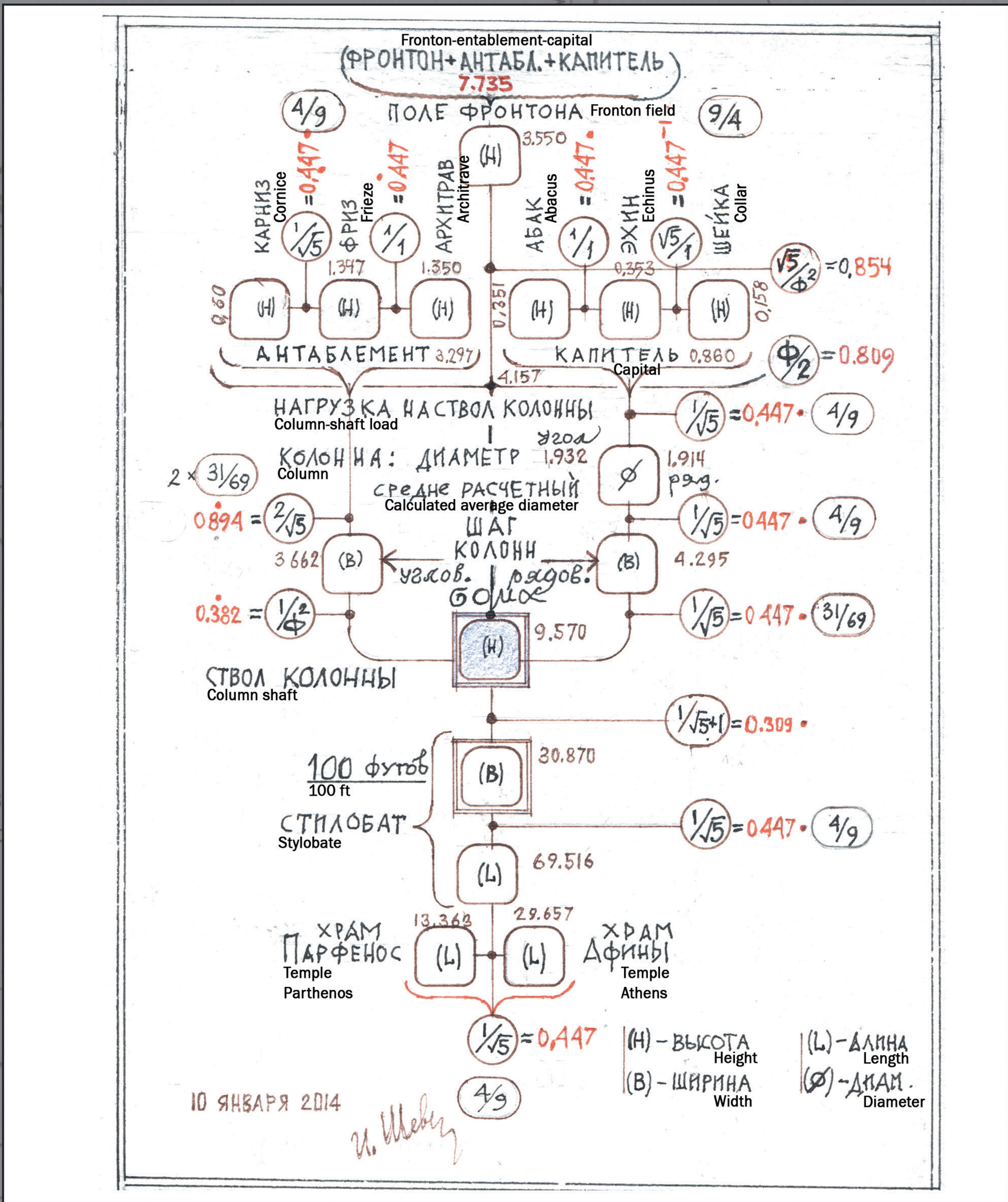
Примечание:

Знак диез (“N) слева вверху от буквы означает удвоение контраста.
Отношение сторон a/b уступило место отношению a/2b.
Знак бемоль (N‹‹) справа внизу от буквы означает уменьшение этого контраста вдвое.
Отношение сторон a/b уступило место отношению 2a/b.
Третью строку заполняют только прямоугольники золотого сечения **Ф** и квадраты **D**.
Прямоугольникам таблицы соответствуют, в таблице №1
(алфавит зрительных образов), строки:
D -1, M - 3, Ф‹‹- 4, S - 6, Ф - 7, L - 8, “D - 10, “M - 12.

Рисунок
Figure 17.1

Рис Точки W_{ϕ} и $W_{\sqrt{\phi}}$.

Совершенная симметрия; отсутствуют связи, представленные числами НР,



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