education research in a climate of peer discussion, with the help of highly qualified experts with varying fields of expertise. This event was an opportunity for young researchers to present their ideas, theoretical difficulties, methodological problems and preliminary research results, in order to get suggestions from other participants and experts about possible developments and different perspectives, opening the way to possible connections with other research projects and cooperation with researchers in other countries.

The participants included PhD students and post-doctoral researchers in mathematics education and others entering mathematics education research from European countries and neighbouring countries. There were 123 applicants, showing the interest raised by this ERME activity, and 73 were finally selected from Germany (18), Portugal (15), France, Italy and Norway (5), Turkey and UK (3), Cyprus, Greece, Israel and Sweden (2) and Algeria, Brazil, Canada, Czech Republic, Denmark, Ghana, Iceland, Libanon, Lybia, Spain and USA (1). The participants presented papers according to the situation of their studies and research work. This could include comprehensive information concerning personal graduate studies and/or research plans; presentations of research work in progress (goals, theoretical framework and methodology); or presentations of preliminary results (with essential information about their goals, theoretical framework and methodology).

The topics of the summer school were: Teacher knowledge and practice; Teacher education and professional development; Teaching and learning advanced mathematics; Cognitive and affective factors in learning and teaching mathematics; Theoretical perspectives, modelling and linguistic and representational aspects of teaching and learning mathematics. The scientific staff was composed of six leading experts in mathematics education: Ferdinando Arzarello, Markku Hannula, Barbara Jaworski, Maria Alessandra Mariotti, João Pedro da Ponte and Heinz Steinbring, and contributed to the Summer School by giving lectures and coordinating the working groups. Moreover, discussion groups were led by Paolo Boero and Rita Borromeo Ferri and there was a discussion devoted to issues that are relevant to the YERME organisation. The evaluation of the summer school was made by Paolo Boero. A complementary social program included a Fado night, a Brazilian night, a cultural session on mathematical competitions, an excursion to the old city of Tavira and a final outdoor night with opera and Portuguese music.

The drive for summer schools came from the spontaneous aggregation of young researchers of different countries at the CERME-II (2001) and CERME-III (2003) conferences. The aim was to create a cooperative style of working and a support to the development of professional preparation and careers in the field of mathematics education. Former YERME summer schools took place in Klagenfurt (Austria, 2002), Podebrady (Czech Republic, 2004), Jyväskylä (Finland, 2006), Trabzon (Turkey, 2008) and Palermo (Italy, 2010). This summer school took place at the University of Algarve in Faro (http://www.ualg.pt/), Campus da Penha, located in the South of Portugal. The organising committee included Ferdinando Arzarello and João Pedro da Ponte (ERME board representatives), Cláudia Canha Nunes and António Guerreiro (local group team representatives) and Paolo Boero (scientific coordinator). The local organisation was based at the Instituto de Educação da Universidade de Lisboa (João Pedro da Ponte, Cláudia Nunes and Marisa Quaresma) and the Escola Superior de Educação e Comunicação da Universidade do Algarve (António Guerreiro, Luciano Veia, Cristolinda Costa and Sandra Nobre).

Models and Modelling in Mathematics Education

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Introduction and background

A major reason why mathematics is the world's single largest educational subject is the fact that mathematics is applied in a multitude of different ways in a huge variety of extra-mathematical subjects, fields and practice areas. Every time mathematics is used to deal with issues, problems, situations and contexts in domains outside of mathematics, mathematical models and modelling are necessarily involved, be it implicitly or explicitly. We begin by giving a brief outline of the basic concepts and terms of mathematical models and modelling before moving on to their educational aspects.

Consider some extra-mathematical domain and imagine that, for one reason or another, we want to come to grips with certain elements, features, phenomena, relationships, properties, issues, problems or questions pertaining to that domain, and that we intend to employ mathematics to do so. We then have to select, from the domain, those objects, relationships, phenomena, questions, etc. which we deem significant for our purpose. Each of the entities thus selected have to be represented by mathematical entities within some realm of mathematics which we reckon to be of relevance in the context. In other words, we map (translate) selected entities, including questions, from the extra-mathematical domain under consideration into mathematical entities belonging and referring to the mathematical realm which has been chosen. The very point of involving mathematics is to seek mathematical answers (by mathematical means) to the translated questions in the mathematical realm and then translate the answers back into the extra-mathematical domain and interpret and evaluate them as answers to the extra-mathematical questions posed at the outset. This process, taking a point of departure in some extra-mathematical domain, moving into some mathematical realm so as to obtain mathematical conclusions and translating these back to the extra-mathematical domain, is known in the literature as the *modelling cycle* (see, for example, Niss, Blum and Galbraith, 2007, p. 4). It is important to keep in mind that building a mathematical model unavoidably involves deliberately and consciously ignoring lots of information, features, facts and circumstances that are judged to have minor importance in relation to the purpose and the context at issue. In other words, modelling oftentimes implies substantial simplification, stylisation, reduction of complexity, etc.

Against this background, a *mathematical model* can be defined in terms of an extra-mathematical domain, D, a mathematical realm, M, and a mapping (translation), f, from D to M. Metaphorically, we can then think of a mathematical model as the triple (D,f,M), which indicates that each of D, f and M is an indispensable component of the model. Sometimes f is also called a "mathematisation" of D by means of M. The use of the set-theoretical metaphor (D,f,M) should not be over-interpreted, since D and M are not only meant to be "sets" consisting of objects (elements) but are also collections of relationships, phenomena, questions (and possible answers) and suchlike, and since f not only operates on objects but also on the relationships, phenomena and questions selected to be the focus of our attention.

Let us illustrate these considerations with a simple example. If we want to decide which of two taxi companies, T and U, with different tariff schemes to choose for a taxi ride from A to B, the extra-mathematical domain (D) consists of taxi rides taking place in a topographical and commercial environment. Depending on the specific setting, significant entities include routes, distances, zones, neighbourhoods, time (of the ride, including waiting time, of the day, of the season, etc.), rates and money, whereas comfort and safety may not be deemed significant if the two taxi companies do not differ in those respects. Assume that we want to choose between T and U solely based on the cost of the rides and that the cost turns out to be determined by the zone location of A and B, the distance between them, and distance and zone dependent rate schemes used by T and U, respectively. Then a suitable mathematical realm (M) to represent the context and situation could consist of real functions, more specifically non-negative, piecewise linear functions defined on the non-negative reals, where the independent variable represents distance travelled and the dependent variable represents cost. The mapping f then specifies the exact form of two functions, one for each company, and translates the questions from D into questions concerning M. Choosing between the two companies would then amount to (a) determining the intervals in which one cost function exceeds the other and identifying the representation of the desired ride in one of these intervals, which leads to a mathematical conclusion of which function or functions have the lower value, and (b) translating this answer back to an answer saying "company T / U should be chosen for this ride" or "it doesn't matter which company you choose for this ride".

When a mathematical model is introduced (selected, modified or constructed) from scratch to deal with aspects of an extra-mathematical context and situation, we say that *mathematical modelling* is taking place. A person who from scratch introduces a model into a context is a *mathematical modeller* for that context. Sometimes a mathematical model is already present in a given context because it has been introduced by others. If so, we often speak of *an application* of mathematics. A person who investigates or assesses such a model may be called a *model analyst* for that context.

The purpose, place and role of models and modelling

Since the late 1960s a growing community of mathematics educators have cultivated an interest in the purpose, place and role of mathematical applications, models and modelling in the teaching and learning of mathematics. This interest is based on two different but certainly compatible ideas. The first idea – of which "mathematics for applications, models and modelling" could be a slogan – is that the utilisation of mathematics in extra-mathematical contexts for extra-mathematical purposes is, in itself, an important activity and endeavour. Thus it should be a primary goal and task of mathematics education to enable students at various levels to engage in such activities. The second idea – sloganized as "applications, models and modelling for the learning of mathematics" – is that dealing with the activation of mathematics in extra-mathematical contexts for extra-mathematical purposes can foster motivation with (some) students for the study of mathematics and help support and consolidate their concept formation, sense-making and experience of meaning in and of mathematics. Thus, the teaching and learning of mathematics for its own sake can take advantage of applications, models and modelling. This is the case, for instance, with the so-called "realistic mathematics education" approach, taken by the Freudenthal Institute in the Netherlands. (For a more detailed account of these ideas, see Blum & Niss, 1991. The two "philosophies" are still significant within the field; see, for instance, Gravemeijer, 2007, and Lesh & Doerr, 2003.)

For a couple of decades mathematics educators working in this area focused on designing and implementing teaching plans and activities on applications, models and modelling, either as part of existing courses, curricula or programmes, or as entirely new teaching units, courses, curricula or programmes. To support all this, teaching materials, including textbooks, and assessment schemes were developed as well. Ideas, views and, above all, experiences were presented and analysed in journals and books and were exchanged and discussed in conferences, such as the International Congresses on Mathematical Instructions (ICMEs) and particularly in the International Conferences on the Teaching of Mathematical Modelling and Applications (ICTMAs), inaugurated in the UK in 1983 and held biennially since then. The community which evolved around these conferences (the International Community of Teachers of Mathematical Modelling and Applications, also with ICTMA as its acronym) was officially established as an Affiliated Study Group of the International Commission on Mathematical Instruction (ICMI) in 2003. For a historical account of the conferences and the community, see Houston, Galbraith & Kaiser, 2008. For an account of the state of the art in the field, see Blum, Galbraith, Henn & Niss, 2007.

From the 1990s onwards, a large body of empirical research has been undertaken in order to investigate a variety of questions concerning the teaching and learning of models and modelling. In the remainder of this article we shall briefly outline a few of the most significant outcomes of this research.

Selected solid findings

In the attempts during at least four decades to attribute a sizable place and role to models and modelling in different mathematics curricula and in different contexts of teaching and learning, two manifest observations emerged again and again. Later on these observations became supported by empirical research to such an extent that they have developed into solid findings of mathematics education research.

The *first* observation and finding is this: while knowledge of and skills in "pure" mathematics are, of course, necessary for an individual's ability to deal with models and to perform modelling, such knowledge and skills are far from sufficient for that undertaking. In other words, there is no guaranteed transfer from mathematical knowledge and skills to knowledge and skills concerning models and modelling. The literature contains many examples of students with a very good knowledge and skills base in mathematics who are not able (without specific teaching) to put their knowledge and skills to use in models and modelling contexts. One reason for this is that all the assumptions, simplifications and decisions, which usually have to be made in order to model a situation or context, involve considering and dealing with matters belonging to the extra-mathematical domain to be modelled. Moreover, it may be necessary to procure extra-mathematical facts, collect data or make measurements. All these things have to be handled on other than purely mathematical grounds. Some students – and some mathematics teachers, too – are unable, reluctant or unwilling to leave their mathematical quarters and do what it takes to engage with extra-mathematical matters while activating their mathematical knowledge and skills. Research publications underpinning this finding include Ikeda & Stephens (1998), Stillman (2002) and Kaiser & Maass (2007).

This first finding suggests that engaging in models and modelling has to be learnt in some way or another, but how can this take place? The big question then is whether it is possible to teach models and modelling in an effective manner so as to generate learning with students and, if so, under what conditions? Leaving aside for a moment the ensuing key questions of what learning of models and modelling means and of how we can recognise learning when it is present, we turn to the *second* observation and finding: the good news is that models and modelling can in fact be taught effectively so as to be learnt by students at various levels, but this requires investments and efforts in terms both of careful and focused design and of teaching and learning environments and activities, and in terms of sufficient time for the activities designed to unfold. It is part of this finding that genuine modelling skill cannot be developed with students by way of teaching focusing on stylised and stereotypical examples in the hope that this will result in transfer to real modelling situations and tasks. Research publications leading to the second solid finding include Ottesen (2001), Maass (2004), Blomhøj & Kjeldsen (2006) and Verschaffel et al. (1999).

We shall return to the questions set aside above. What does learning of models and modelling mean and how can we recognise it when we encounter it? Here, the so-called modelling competencies form the essential component. Theoretical and empirical research shows that being able to do modelling amounts to being able to successfully undertake a series of competencies called upon in the modelling cycle (see, for example, Blomhøj & Jensen, 2003, and Maass, 2006). A set of solid findings identifies the difficulties and challenges embedded in the set of modelling competencies, some of which give rise to obstacles to coming to grips with models and modelling (Galbraith & Stillman, 2006), whilst others put forward effective means for overcoming these difficulties and meeting these challenges. For example, the effects of the context of modelling tasks with an equivalent mathematical content can be dramatic, for better and for worse (Busse & Kaiser, 2003, Busse, 2011, and Stillman, 2000).

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Grading Mathematics Education Research Journals

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Presentation of the project and initial motives

Nowadays, all researchers are aware of the increasing importance accorded to the ranking and grading of scientific journals; it is now difficult to escape their influence. The systems that currently exist are often based on crude statistical analyses that have little to do with scientific quality (see, for example, Arnold & Fowler 2011). For these reasons, the Education Committee of the European Mathematical Society (EMS), together with the Executive Committee of the European Society for Research in Mathematics Education (ERME) and supported by the International Commission for Mathematical Instruction (ICMI), decided in 2011 to organise a consultation in order to propose a grading of research journals in mathematics education based on expert judgment. A similar project has already been carried out for chemical education and science education journals (Towns & Kraft, 2011).

The approach adopted was to initiate a process which will need further elaboration and regular updating. For this reason, amongst many possible choices of method, we always opted for what appeared to be the most straightforward. We present below our methods and the results obtained.

Organisation of grading by experts

A working group, bringing together members of the ERME board and members of the EMS educational committee, was formed to take charge of the whole process. We (the members of this group) first prepared a long list comprising 49 journals. We graded the journals and compared our grades with the European Reference Index for the Humanities 2011 lists (https://www2.esf.org/asp/ERIH/Foreword/search.asp). This led us to retain a shortlist of 28 journals (all the mathematics education research journals mentioned as international on the ERIH list have been kept).

At the same time we constituted a panel of 91 experts in the field, representing the 42 countries members of the EMS and the ERME. Each country was represented by one to seven experts, according to the size of the mathematics education research community in each country.

These experts were contacted and asked to grade the journals, using the scale presented below. They were also invited to formulate any comments they wished to make on the process and to suggest other journal titles if they considered that important journals were missing from the list.